Online Appendix to:
Marginalized Predictive Likelihood Comparisons of Linear Gaussian State-Space Models with Applications to DSGE, DSGE-VAR, and VAR Models

Anders Warne, Günter Coenen and Kai Christoffel

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Appendix A: Posterior Properties of the BVAR Model

The BVAR is estimated with the methodology suggested in Bańbura, Giannone and Reichlin (2010) and therefore relies on using dummy observations when implementing the normal-inverted Wishart version of the Minnesota prior. Below we will first present the prior and posterior distribution and thereafter show the relation between the prior parameters and the T_d dummy observations; see also Lubik and Schorfheide (2006).

The VAR representation of y_t is given in equation (9) of the paper, with $\epsilon_t \sim N_n(0,\Omega)$. Let $X_t = [1\ y_{t-1}' \cdots y_{t-p}']'$ be an $(1+np)$-dimensional vector, while the $n \times (1+np)$ matrix $\Phi = [\Phi_0\ \Phi_1 \cdots \Phi_p]$. This means that the VAR model can be expressed as

$$y_t = \Phi X_t + \epsilon_t, \quad t = 1,\dots,T. \tag{A.1}$$

Stacking the VAR system as $y = [y_1 \cdots y_T]$, $X = [X_1 \cdots X_T]$, and $\epsilon = [\epsilon_1 \cdots \epsilon_T]$, the log-likelihood is given by

$$\log p\bigl(y\big|X_1;\Phi,\Omega\bigr) = -\frac{nT}{2}\log(2\pi) - \frac{T}{2}\log\bigl|\Omega\bigr| - \frac{1}{2}\mathrm{tr}\bigl[\Omega^{-1}\epsilon\epsilon'\bigr], \tag{A.2}$$

where, for convenience, we use the same notation for the random variables as their realizations.

The normal-inverted Wishart prior for (Φ,Ω) is given by

$$\mathrm{vec}\bigl(\Phi\bigr)\big|\Omega \sim N_{n(np+1)}\bigl(\mathrm{vec}\bigl(\Phi_\mu\bigr),\bigl[\Omega_\Phi \otimes \Omega\bigr]\bigr), \tag{A.3}$$

$$\Omega \sim IW_n\bigl(A,v\bigr). \tag{A.4}$$

This means that the sum of the log-likelihood and the log prior is given by

$$\begin{aligned}\log p\bigl(y,\Phi,\Omega\big|X_1\bigr) = &-\frac{n(T+np+1)}{2}\log(2\pi) - \frac{nv}{2}\log(2) - \frac{n(n-1)}{4}\log(\pi)\\ &-\log\Gamma_n(v) - \frac{n}{2}\log\bigl|\Omega_\Phi\bigr| + \frac{v}{2}\log\bigl|A\bigr|\\ &-\frac{T+n(p+1)+v+2}{2}\log\bigl|\Omega\bigr|\\ &-\frac{1}{2}\mathrm{tr}\left[\Omega^{-1}\left(\epsilon\epsilon' + A + \bigl(\Phi-\Phi_\mu\bigr)\Omega_\Phi^{-1}\bigl(\Phi-\Phi_\mu\bigr)'\right)\right].\end{aligned} \tag{A.5}$$

Using standard "Zellner" algebra, it is straightforward to show that

$$\epsilon\epsilon' + A + (\Phi - \Phi_\mu)\Omega_\Phi^{-1}(\Phi - \Phi_\mu)') = \left(\Phi - \bar{\Phi}\right)\left(XX' + \Omega_\Phi^{-1}\right)\left(\Phi - \bar{\Phi}\right)' + S, \tag{A.6}$$

where

$$\bar{\Phi} = \left(yX' + \Phi_\mu\Omega_\Phi^{-1}\right)\left(XX' + \Omega_\Phi^{-1}\right)^{-1},$$
$$S = yy' + A + \Phi_\mu\Omega_\Phi^{-1}\Phi_\mu' - \bar{\Phi}\left(XX' + \Omega_\Phi^{-1}\right)\bar{\Phi}'.$$

Substituting for (A.6) in (A.5), we find that the conjugate normal-inverted Wishart prior gives us a normal posterior for $\Phi|\Omega$ and an inverted Wishart marginal posterior of Ω. Specifically,

$$\text{vec}\big(\Phi\big)\big|\Omega, y, X_1 \sim N_{n(np+1)}\big(\text{vec}\big(\bar{\Phi}\big), \big[(XX' + \Omega_\Phi^{-1})^{-1} \otimes \Omega\big]\big), \tag{A.7}$$

$$\Omega\big|y, X_1 \sim IW_n\big(S, T + v\big). \tag{A.8}$$

Combining these posterior results with equations (A.5) and (A.6) it follows that the log marginal likelihood is given by

$$\begin{aligned} \log p\big(y\big|X_1\big) = &-\frac{nT}{2}\log(\pi) + \log\Gamma_n(T+v) - \log\Gamma_n(v) - \frac{n}{2}\log\left|\Omega_\Phi\right| \\ &+ \frac{v}{2}\log\left|A\right| - \frac{n}{2}\log\left|XX' + \Omega_\Phi^{-1}\right| - \frac{T+v}{2}\log\left|S\right|. \end{aligned} \tag{A.9}$$

The prior in (A.3) and (A.4) can be implemented through $T_d = n(p+2)+1$ dummy observations by prepending the y $(n \times T)$ and X $(np+1 \times T)$ matrices with the following:

$$\begin{aligned} y_{(d)} &= \begin{bmatrix} \lambda_o^{-1}\text{diag}[\delta \odot \omega] & 0_{n\times n(p-1)} & \text{diag}[\omega] & 0_{n\times 1} & \tau^{-1}\text{diag}[\delta \odot \mu] \end{bmatrix} \\ X_{(d)} &= \begin{bmatrix} 0_{1\times np} & 0_{1\times n} & \gamma^{-1} & 0_{1\times n} \\ \lambda_o^{-1}\big(j_p \otimes \text{diag}[\omega]\big) & 0_{np\times n} & 0_{np\times 1} & \tau^{-1}\big(\imath_p \otimes \text{diag}[\mu]\big) \end{bmatrix}. \end{aligned} \tag{A.10}$$

The vector $\imath_p$ is a p-dimension unit vector, while the $p \times p$ matrix $j_p = \text{diag}[1 \ \cdots \ p]$. The hyperparameter $\lambda_o > 0$ gives the overall tightness in the Minnesota prior, the cross-equation tightness is set to unity, while the harmonc lag decay hyperparameter is equal to 2. The hyperparameter $\tau > 0$ handles shrinkage for the sum of cofficients prior on $(I_n - \sum_{i=1}^{p}\Phi_i)$, where $\tau \to 0$ means that the prior on the sum of the lag cofficients approach the case of exact differences, and where shrinkage decreases as τ becomes larger. The n-dimensional vector δ gives the prior mean of the diagonal of Φ_1, ω is a vector of scale parameters for the residuals ϵ_{it}, while μ is a vector that reflects the mean of y_{it}. Finally, γ reflects the overall tightness on Φ_0.

The relationship between the dummy observations and the prior parameters $(\Phi_\mu, \Omega_\Phi, A, v)$ are:

$$\Phi_\mu = y_{(d)}X_{(d)}'\left(X_{(d)}X_{(d)}'\right)^{-1}, \qquad \Omega_\Phi = \left(X_{(d)}X_{(d)}'\right)^{-1},$$
$$A = \left(y_{(d)} - \Phi_\mu X_{(d)}\right)\left(y_{(d)} - \Phi_\mu X_{(d)}\right)', \qquad v = T_d - (np+1) + 2.$$

This guarantees that the prior mean of Ω exists. Letting $y_\star = [y_{(d)}\, y]$ and $X_\star = [X_{(d)}\, X]$, it follows that the posterior parameters

$$\begin{aligned}\bar{\Phi} &= y_\star X_\star' \big(X_\star X_\star'\big)^{-1}, \\ XX' + \Omega_{\bar{\Phi}}^{-1} &= X_\star X_\star', \\ S &= \big(y_\star - \bar{\Phi} X_\star\big)\big(y_\star - \bar{\Phi} X_\star\big)'.\end{aligned}$$

In the empirical application, $\tau = 10\lambda_o$, i.e. a relatively loose prior on the sum of the autoregressive matrices. The hyperparameters $\delta_i = 0$ if y_{it} is a first differenced variable and $\delta_i = 1$ when y_{it} is a levels variable. The scale parameters ω_i is given by the within-sample residual standard deviation from an AR(p) model for y_{it}, while μ_i is equal to the within-sample mean of y_{it}. The parameter $\varsigma = \gamma^{-1}$ is set to a very small number, which takes care of having an improper prior on Φ_0.

The formula suggested by Bańbura et al. (2010) for selecting λ_o can be expressed as

$$\bar{\lambda}_o(\phi) = \arg\min_{\lambda_o} \left| \phi - \frac{1}{q} \sum_{j=1}^{q} \frac{\sigma_j^2(\lambda_o)}{\sigma_j^2(0)} \right|,$$

where $\phi \in (0,1)$ is the desired fit, and $\sigma_j^2(\tilde{\lambda}_o)$ is the one-step-ahead mean square forecast error of variable j when $\lambda_o = \tilde{\lambda}_o$. The one-step-ahead within-sample mean square forecast errors used in the selection scheme are based on the sample 1985Q1–1998Q4. With $\phi = 0.5$, $q = 3$ using real GDP growth, the GDP deflator, and the short-term nominal interest rate, this selection scheme sets $\bar{\lambda}_o = 0.0693$ when $p = 4$.

It should be noted that having an improper prior on Φ_0 technically means that Ω_Φ is singular. This needs to be taken into account when computing, e.g., the log marginal likelihood in (A.9). To deal with this, let

$$X = \begin{bmatrix} \imath_T' \\ Y \end{bmatrix}, \qquad X_{(d)} = \begin{bmatrix} 0_{1\times T_d} \\ Y_{(d)} \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \Phi_1 & \dots & \Phi_p \end{bmatrix}, \qquad \Omega_\Phi = \begin{bmatrix} \gamma^2 & 0_{1\times np} \\ 0_{np\times 1} & \Omega_\Gamma \end{bmatrix},$$

where $\imath_T$ is a $T \times 1$ unit vector. The prior for the BVAR is now expressed as

$$\mathrm{vec}\big(\Gamma\big)\big|\Omega \sim N_{n^2p}\big(\mathrm{vec}\big(\Gamma_\mu\big), \big[\Omega_\Gamma \otimes \Omega\big]\big), \tag{A.11}$$

while $p(\Phi_0) = 1$ and the prior of Ω is given by (A.4). Let $Z = y - \Gamma Y$, $\bar{\Phi}_0 = T^{-1} Z \imath_T$, and let

$$D = I_T - T^{-1}\imath_T \imath_T',$$

a $T \times T$ symmetric and idempotent matrix. Through the usual Zellner algebra we have that

$$\epsilon\epsilon' = ZDZ' + \big(\Phi_0 - \bar{\Phi}_0\big)\imath_T'\imath_T\big(\Phi_0 - \bar{\Phi}_0\big)'.$$

Furthermore, with D being symmetric and idempotent we may define $\tilde{Z} = ZD$, such that $\tilde{y} = yD$, $\tilde{Y} = YD$ and $ZDZ' = \tilde{Z}\tilde{Z}'$. The Zellner algebra now provides us with

$$\tilde{Z}\tilde{Z}' + (\Gamma - \Gamma_\mu)\Omega_\Gamma^{-1}(\Gamma - \Gamma_\mu)' + A = (\Gamma - \bar{\Gamma})\left(\tilde{Y}\tilde{Y}' + \Omega_\Gamma^{-1}\right)(\Gamma - \bar{\Gamma})' + S,$$

where

$$\bar{\Gamma} = \left(\tilde{y}\tilde{Y}' + \Gamma_\mu\Omega_\Gamma^{-1}\right)\left(\tilde{Y}\tilde{Y}' + \Omega_\Gamma^{-1}\right)^{-1}$$
$$S = \tilde{y}\tilde{y}' + A + \Gamma_\mu\Omega_\Gamma^{-1}\Gamma_\mu' - \bar{\Gamma}\left(\tilde{Y}\tilde{Y}' + \Omega_\Gamma^{-1}\right)\bar{\Gamma}'.$$

It can therefore be shown that the normal-inverted Wishart posterior for the VAR parameters is given by

$$\Phi_0 \big| \Gamma, \Omega, y, X_1 \sim N_n\big(\bar{\Phi}_0, T^{-1}\Omega\big), \tag{A.12}$$
$$\mathrm{vec}(\Gamma) \big| \Omega, y, X_1 \sim N_{n^2p}\big(\mathrm{vec}(\bar{\Gamma}), \big[(\tilde{Y}\tilde{Y}' + \Omega_\Gamma^{-1})^{-1} \otimes \Omega\big]\big) \tag{A.13}$$
$$\Omega \big| y, X_1 \sim IW_n\big(S, T + v - 1\big). \tag{A.14}$$

Hence, the improper prior on Φ_0 results in a loss of degrees of freedom for the posterior of Ω. Furthermore, the log marginal likelihood is

$$\begin{aligned}\log p\big(y \big| X_1\big) = &-\frac{n(T-1)}{2}\log(\pi) + \log\Gamma_n(T+v-1) - \log\Gamma_n(v) - \frac{n}{2}\log\big|\Omega_\Gamma\big| \\ &+ \frac{v}{2}\log\big|A\big| - \frac{n}{2}\log(T) - \frac{n}{2}\log\big|\tilde{Y}\tilde{Y}' + \Omega_\Gamma^{-1}\big| - \frac{T+v-1}{2}\log\big|S\big|,\end{aligned} \tag{A.15}$$

where the term $\log(T)$ stems from $T = \imath_T'\imath_T$ and is obtained when integrating out Φ_0 from the joint posterior. The relationship between the dummy observations and the prior parameters is

$$\Gamma_\mu = y_{(d)}Y_{(d)}'\left(Y_{(d)}Y_{(d)}'\right)^{-1}, \qquad \Omega_\Gamma = \left(Y_{(d)}Y_{(d)}'\right)^{-1},$$
$$A = \left(y_{(d)} - \Gamma_\mu Y_{(d)}\right)\left(y_{(d)} - \Gamma_\mu Y_{(d)}\right)', \qquad v = T_d - (np+1) + 2.$$

Letting $\tilde{y}_\star = [y_{(d)}\ \tilde{y}]$ and $\tilde{Y}_\star = [Y_{(d)}\ \tilde{Y}]$, it follows that the posterior parameters

$$\bar{\Gamma} = \tilde{y}_\star\tilde{Y}_\star'\big(\tilde{Y}_\star\tilde{Y}_\star'\big)^{-1},$$
$$\tilde{Y}\tilde{Y}' + \Omega_\Gamma^{-1} = \tilde{Y}_\star\tilde{Y}_\star',$$
$$S = \big(\tilde{y}_\star - \bar{\Gamma}\tilde{Y}_\star\big)\big(\tilde{y}_\star - \bar{\Gamma}\tilde{Y}_\star\big)'.$$

Appendix B: Posterior Properties of the Random Walk Model

The purpose of this Appendix is to provide technical details on the predictive density of the random walk model with a standard diffuse prior on the residual covariance matrix. An analytical expression of the predictive density is derived and its mean vector and covariance matrix are also determined.

To these ends, let

$$y_t = y_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{B.1}$$

where the residuals ε_t are assumed to be i.i.d. $N(0, \Omega)$ with Ω positive definite and y_0 is fixed. The diffuse prior is given by

$$p(\Omega) \propto \left|\Omega\right|^{-(n+1)/2}. \tag{B.2}$$

Stacking the model in (B.1) into $n \times T$ matrices $y = [y_1 \cdots y_T]$ and $\varepsilon = [\varepsilon_1 \cdots \varepsilon_T]$, with the realized values, for convenience, being denoted the same way, the posterior distribution is proportional to the prior times the likelihood, which in natural logarithms can be expressed as

$$\log p\big(y\big|y_0, \Omega\big) + \log p(\Omega) = -\frac{nT}{2}\log(2\pi) - \frac{T+n+1}{2}\log\left|\Omega\right| - \frac{1}{2}\mathrm{tr}\big[\Omega^{-1}\varepsilon\varepsilon'\big]. \tag{B.3}$$

Recognizing that the last two terms on the right hand side of (B.3) form the log of the kernel of the n-dimensional inverted Wishart distribution with location matrix $\varepsilon\varepsilon'$ and T degrees of freedom, we obtain

$$\begin{aligned}\log p\big(\Omega\big|y, y_0\big) = &-\frac{nT}{2}\log(2) - \frac{n(n-1)}{4}\log(\pi) - \log\Gamma_n(T) + \frac{T}{2}\log\left|\varepsilon\varepsilon'\right| \\ &- \frac{T+n+1}{2}\log\left|\Omega\right| - \frac{1}{2}\mathrm{tr}\big[\Omega^{-1}\varepsilon\varepsilon'\big],\end{aligned} \tag{B.4}$$

where

$$\Gamma_n(T) = \prod_{i=1}^{n} \Gamma([T-i+1]/2),$$

for $T \geq n > 0$ with $\Gamma(\cdot)$ being the gamma function. From Bayes theorem it therefore follows that the log marginal likelihood is given by (B.3) minus (B.4), i.e.

$$\log p\big(y\big|y_0\big) = -\frac{n(2T-n+1)}{4}\log(\pi) + \log\Gamma_n(T) - \frac{T}{2}\log\left|\varepsilon\varepsilon'\right|. \tag{B.5}$$

Normal Approximation of Marginal Predictive Likelihood

When forecasting with the random walk model it holds that

$$E\big[y_{T+h}\big|y, y_0, \Omega\big] = y_T, \quad h = 1, \ldots, h^*. \tag{B.6}$$

The forecast error is therefore equal to the accumulation of ε_{T+i} over $i = 1, \ldots, h$, while the forecast error covariance matrix given Ω is

$$C\big(y_{T+h}\big|y, y_0, \Omega\big) = h\Omega, \quad h = 1, \ldots, h^*. \tag{B.7}$$

It is well-known that the covariance matrix $C(y_{T+h}|y, y_0)$ is equal to the mean of the covariance matrix in (B.7) with respect to the posterior of Ω plus the covariance matrix of the deviation of the mean in (B.6) and its population mean $E[y_{T+h}|y, y_0]$. The latter term is zero since the population mean is also y_T, while the former term is given by h times the mean of the posterior of Ω.[1] That is,

$$C\bigl(y_{T+h}|y, y_0\bigr) = \frac{h}{T-n-1}\varepsilon\varepsilon'. \tag{B.8}$$

When computing the marginal predictive likelihood with a normal approximation for the full system we therefore make use of the realized forecast errors $y_{T+h} - y_T$ and the covariance matrix in (B.8).

When forecasting only a subset of the variables we need to take into account how the posterior distribution for the covariance matrix of the corresponding subset of residuals is related to the posterior $p(\Omega|y, y_0)$. Let S be an $n \times n_h$ matrix of columns from I_n which selects $y_{s,T+h} = S'y_{T+h}$.[2] Similarly, let $S_\perp$ the the $n \times (n - n_h)$ matrix which selects the remaining variables from the y_{T+h} vector. Define

$$M = \bigl[S \;\; S_\perp\bigr], \tag{B.9}$$

i.e. M is an $n \times n$ matrix made up of all the columns of the identity matrix and therefore has a unit determinant while $M^{-1} = M'$. The posterior distribution of $\Omega_M = M'\Omega M$ is an n-dimensional inverted Wishart with location matrix $M'\varepsilon\varepsilon'M$ and T degrees of freedom. Letting $\Omega_S = S'\Omega S$, it follows from, e.g., Bauwens, Lubrano and Richard (1999, Theorem A.17) that the posterior of Ω_S is an n_h-dimensional inverted Wishart with location matrix $S'\varepsilon\varepsilon'S$ and $T-n+n_h$ degrees of freedom.

With this in mind, the normal approximation of the marginal predictive likelihood for the subset of variables is based on the mean forecast error $y_{s,T+h} - y_{s,T}$ and the population covariance matrix

$$C\bigl(y_{s,T+h}|y, y_0\bigr) = \frac{h}{T-n-1}S'\varepsilon\varepsilon'S. \tag{B.10}$$

Analytical Form of the Marginal Predictive Likelihood

The determination of the marginal predictive likelihood requires an expression for the conditional likelihood function $p(y_{s,T+h}|y, y_0, \Omega)$. Making use of $y_{T+h|T} = y_T$ and $\Sigma_{y,T+h|T} = h\Omega$ we find that the marginalized conditional log-likelihood for the random walk model is given by

$$\log p\bigl(y_{s,T+h}|y, y_0; \Omega\bigr) = -\frac{n_h}{2}\log(2\pi h) - \frac{1}{2}\log\bigl|\Omega_S\bigr| - \frac{1}{2h}\mathrm{tr}\bigl[\Omega_S^{-1}\varepsilon_{s,T,h}\varepsilon_{s,T,h}'\bigr], \tag{B.11}$$

where $\varepsilon_{s,T,h} = y_{s,T+h} - y_{s,T}$, and the term involving $\log(h)$ is due to $|h\Omega_S| = h^{n_h}|\Omega_S|$.

[1] More generally, the posterior distribution of $h\Omega$ is inverted Wishart with location parameter $h\varepsilon\varepsilon'$ and T degrees of freedom.

[2] We here suppress the subscript h from S_h.

The product of the conditional likelihood of $y_{s,T+h}$ and the posterior of Ω_S is given by:

$$p\big(y_{s,T+h}, \Omega_S \big| y, y_0\big) = \frac{\big|S'\varepsilon\varepsilon' S\big|^{(T-n+n_h)/2}}{(2\pi h)^{n_h/2} 2^{(T-n+n_h)n_h/2} \pi^{n_h(n_h-1)/4} \Gamma_{n_h}(T-n+n_h)} \times \big|\Omega_S\big|^{-(T-n+2n_h+2)/2} \times \exp\left[-\frac{1}{2}\text{tr}\left(\Omega_S^{-1}\left[S'\varepsilon\varepsilon' S + h^{-1}\varepsilon_{s,T,h}\varepsilon_{s,T,h}'\right]\right)\right]. \quad \text{(B.12)}$$

Recognizing that the two terms involving Ω_S is the kernel of an n_h-dimensional inverted Wishart distribution with location matrix $S'\varepsilon\varepsilon' S + h^{-1}\varepsilon_{s,T,h}\varepsilon_{s,T,h}'$ and $T-n+n_h+1$ degrees of freedom, it follows that the integral of the density $p\big(y_{s,T+h}, \Omega_S \big| y, y_0\big)$ with respect to Ω_S is equal to the expression in the first term on the right hand side of equation (B.12) times the inverse of the integration constant of the $IW_{n_h}(S'\varepsilon\varepsilon' S + h^{-1}\varepsilon_{s,T,h}\varepsilon_{s,T,h}', T-n+n_h+1)$ distribution. We therefore find that

$$p\big(y_{s,T+h} \big| y, y_0\big) = \frac{\Gamma_{n_h}(T-n+n_h+1)\big|hS'\varepsilon\varepsilon' S\big|^{-1/2}}{\pi^{n_h/2}\Gamma_{n_h}(T-n+n_h)\big|I_{n_h} + \big(hS'\varepsilon\varepsilon' S\big)^{-1}\varepsilon_{s,T,h}\varepsilon_{s,T,h}'\big|^{(T-n+n_h+1)/2}}. \quad \text{(B.13)}$$

In other words (and as expected), the density of $y_{s,T+h}\big|y, y_0$ is an n_h-dimensional t-distribution with mean $y_{s,T}$, covariance matrix given in (B.10), and $T-n+n_h$ degrees of freedom; see, e.g., Bauwens et al. (1999, Appendix A) for details.[3]

[3]Notice also that $\big|I_{n_h} + \big(hS'\varepsilon\varepsilon' S\big)^{-1}\varepsilon_{s,T,h}\varepsilon_{s,T,h}'\big| = 1 + \varepsilon_{s,T,h}'\big(hS'\varepsilon\varepsilon' S\big)^{-1}\varepsilon_{s,T,h}$; see, e.g., Magnus and Neudecker (1988, Proof of Theorem 1.9).

Appendix C: Marginalization with a Kalman Filter

This section presents the necessary equations for computing the marginalized conditional log-likelihood in linear Gaussian state-space models with a Kalman filter. Let the n-dimensional vector of observable variables y_t be linked to a vector of state variables ξ_t of dimension r through equation

$$y_t = \mu + H'\xi_t + w_t, \quad t = 1, \dots, T. \tag{C.1}$$

The measurement errors, w_t, are assumed to be i.i.d. $N(0, R)$, with R being an $n \times n$ positive semidefinite matrix, while the state variables are determined from a VAR system:

$$\xi_t = F\xi_{t-1} + B\eta_t, \quad t = 1, \dots, T. \tag{C.2}$$

The state shocks, η_t, are of dimension q and i.i.d. $N(0, I_q)$, while F is an $r \times r$ matrix, and B is $r \times q$. The parameters of this model, (μ, H, R, F, B), are all uniquely determined by the vector of parameters θ_m from model m. Provided that $H'\xi_t$ is stationary, the vector μ is the population mean of y_t conditional on θ_m.

In order to handle marginalization, let $\mu_{s,i} = S_i'\mu$, $H_{s,i} = HS_i$, and $R_{s,i} = S_i'RS_i$ when $n_i \geq 1$ for $i = 1, \dots, h$. The selection matrix S_i has dimension $n \times n_i$ with $n_i \in \{0, 1, \dots, n\}$ for $i = 1, \dots, h$, has rank equal to n_i and is known. For example, the columns of S_i are taken from I_n such that n_i unique entries of y_{T+i} are selected.

When $n_i \geq 1$ the one-step-ahead forecasts of $y_{s,T+i}$ and its forecast error covariance matrix conditional on information available at time $T+i-1$ are given by

$$\begin{aligned} y_{s,T+i|T+i-1} &= \mu_{s,i} + H_{s,i}'\xi_{T+i|T+i-1}, \\ \Sigma_{y_s,T+i|T+i-1} &= H_{s,i}'P_{T+i|T+i-1}H_{s,i} + R_{s,i}. \end{aligned}$$

If $n_{i-1} = 0$, the one-step-ahead forecast of the state variables and the corresponding forecast error covariance matrix are

$$\begin{aligned} \xi_{T+i|T+i-1} &= F\xi_{T+i-1|T+i-2} \\ &= F\xi_{T+i-1|T+i-1}, \\ P_{T+i|T+i-1} &= FP_{T+i-1|T+i-2}F' + BB' \\ &= FP_{T+i-1|T+i-1}F' + BB', \end{aligned}$$

while if $n_{i-1} \geq 1$ we instead obtain

$$\begin{aligned}
\xi_{T+i|T+i-1} &= F\xi_{T+i-1|T+i-2} + K_{T+i-1}\bigl(y^o_{s,T+i-1} - y_{s,T+i-1|T+i-2}\bigr) \\
&= F\xi_{T+i-1|T+i-1}, \\
P_{T+i|T+i-1} &= \bigl(F - K_{T+i-1}H'_{s,i-1}\bigr)P_{T+i-1|T+i-2}\bigl(F - K_{T+i-1}H'_{s,i-1}\bigr)' \\
&\quad + K_{T+i-1}R_{s,i-1}K'_{T+i-1} + BB' \\
&= FP_{T+i-1|T+i-1}F' + BB', \\
K_{T+i-1} &= FP_{T+i-1|T+i-2}H_{s,i-1}\Sigma^{-1}_{y_s,T+i-1|T+i-2}.
\end{aligned}$$

The filtering equations are initialized by $\xi_{T|T}$, $P_{T|T}$, and K_T, respectively, obtained from the Kalman filter estimates of the state variables using $\mathcal{Y}^o_T$, while $n_0 = n$ if all entries in Y_T are observed.

for $h \geq 1$, the log of the marginalized conditional likelihood is given by

$$\log p\bigl(\mathcal{Y}^o_{s,T,h}\big|\mathcal{Y}^o_T, \theta_m, m\bigr) = \sum_{i=1}^{h} \log p\bigl(y^o_{s,T+i}\big|\mathcal{Y}^o_{s,T,i-1}, \mathcal{Y}^o_T, \theta_m, m\bigr), \tag{C.3}$$

where $\mathcal{Y}^o_{s,T,0}$ is empty by definition. If $n_i \geq 1$ the marginalized conditional log-likelihood value at $T+i$ is

$$\begin{aligned}
\log p\bigl(y^o_{s,T+i}\big|\mathcal{Y}^o_{s,T,i-1}, \mathcal{Y}^o_T, \theta_m, m\bigr) &= -\frac{n_i}{2}\log(2\pi) - \frac{1}{2}\log\bigl|\Sigma_{y_s,T+i|T+i-1}\bigr| \\
-\frac{1}{2}\left(y^o_{s,T+i} - y_{s,T+i|T+i-1}\right)' \Sigma^{-1}_{y_s,T+i|T+i-1} &\left(y^o_{s,T+i} - y_{s,T+i|T+i-1}\right),
\end{aligned} \tag{C.4}$$

while it is zero when $n_i = 0$. Notice that the Kalman filter based approach generates a *bottom-up* evaluation of the marginalized conditional likelihood.

An interesting special case of the above filtering equations arises when we are concerned with the marginal h-step-ahead forecast of $y_{s,T+h}$. We here have that $n_i = 0$ for $i = 1, \ldots, h-1$, while

$$\begin{aligned}
y_{s,T+h|T} &= \mu_{s,h} + H'_{s,h}F^h\xi_{T|T}, \\
\Sigma_{y_s,T+h|T} &= H'_{s,h}P_{T+h|T}H_{s,h} + R_{s,h}, \\
P_{T+i|T} &= FP_{T+i-1|T}F' + BB', \quad i = 1, \ldots, h.
\end{aligned}$$

The marginalized conditional log-likelihood value at $T+h$ is:

$$\begin{aligned}
\log p\bigl(y^o_{s,T+h}\big|\mathcal{Y}^o_T, \theta_m, m\bigr) &= -\frac{n_h}{2}\log(2\pi) - \frac{1}{2}\log\bigl|\Sigma_{y_s,T+h|T}\bigr| \\
-\frac{1}{2}\left(y^o_{s,T+h} - y_{s,T+h|T}\right)' \Sigma^{-1}_{y_s,T+h|T} &\left(y^o_{s,T+h} - y_{s,T+h|T}\right).
\end{aligned} \tag{C.5}$$

Appendix D: Numerical Precision of the MC Estimates

The precision of the Monte Carlo (MC) integration estimator of the log predictive likelihood (cf. Section 2.2 in the paper) may be assessed with the usual numerical standard error based on within posterior sample information. Posterior sampling of the parameters of the NAWM and of the DSGE-VAR has been conducted with the random walk Metropolis sampler (see, e.g., An and Schorfheide, 2007) and the draws are therefore (highly) correlated, while the BVAR allows for direct sampling so that its parameter draws are independent. Accordingly, the numerical standard errors of the point estimates for the NAWM and the DSGE-VAR can be obtained with the Newey and West (1987) estimator, while those of the BVAR exclude autocorrelation.[4]

We typically find that the standard errors of the log predictive likelihood are the largest for the large selection of the observed variables and the smallest for the small selection. The standard errors of the log predictive likelihood for the NAWM, DSGE-VAR, and the BVAR before 2008Q4 are small and generally below 0.05, also for the eight-quarter-ahead forecasts and the large selection. At the onset of the Great Recession when the log predictive likelihood is small, these standard errors increase, especially for the BVAR model. On average, the standard errors are the smallest for the NAWM and the largest for the BVAR and they increase with the forecast horizon. For instance, the average standard error of the eight-step-ahead forecasts of the NAWM is about 0.02 for the large selection, 0.03 for the DSGE-VAR, and 0.08 for the BVAR.

To further document our findings, Table E.2 reports the point estimates of the log predictive likelihood and the numerical standard errors for the case when the historical data ends in 2007Q4 and the forecasts are performed over the horizon 2008Q1–2009Q4. It is striking that the largest standard errors are obtained at the onset of the Great Recession in 2008Q4 and, in particular, in 2009Q1. The estimated log predictive likelihood values are also the smallest for these quarters. For example, in the case of the large selection the estimated value for the BVAR is approximately -61.2 in 2009Q1 with a standard error of close to unity, which is also the largest recorded standard error over all selections, models, time periods, and forecast horizons. By contrast, the log predictive likelihood of the NAWM is estimated at -40.0 for this quarter, with a standard error of about 0.1. These two predictive likelihood values may be compared with the average log predictive score for five-quarter-ahead forecasts which is -15.7 for the BVAR and -17.9 for the NAWM.

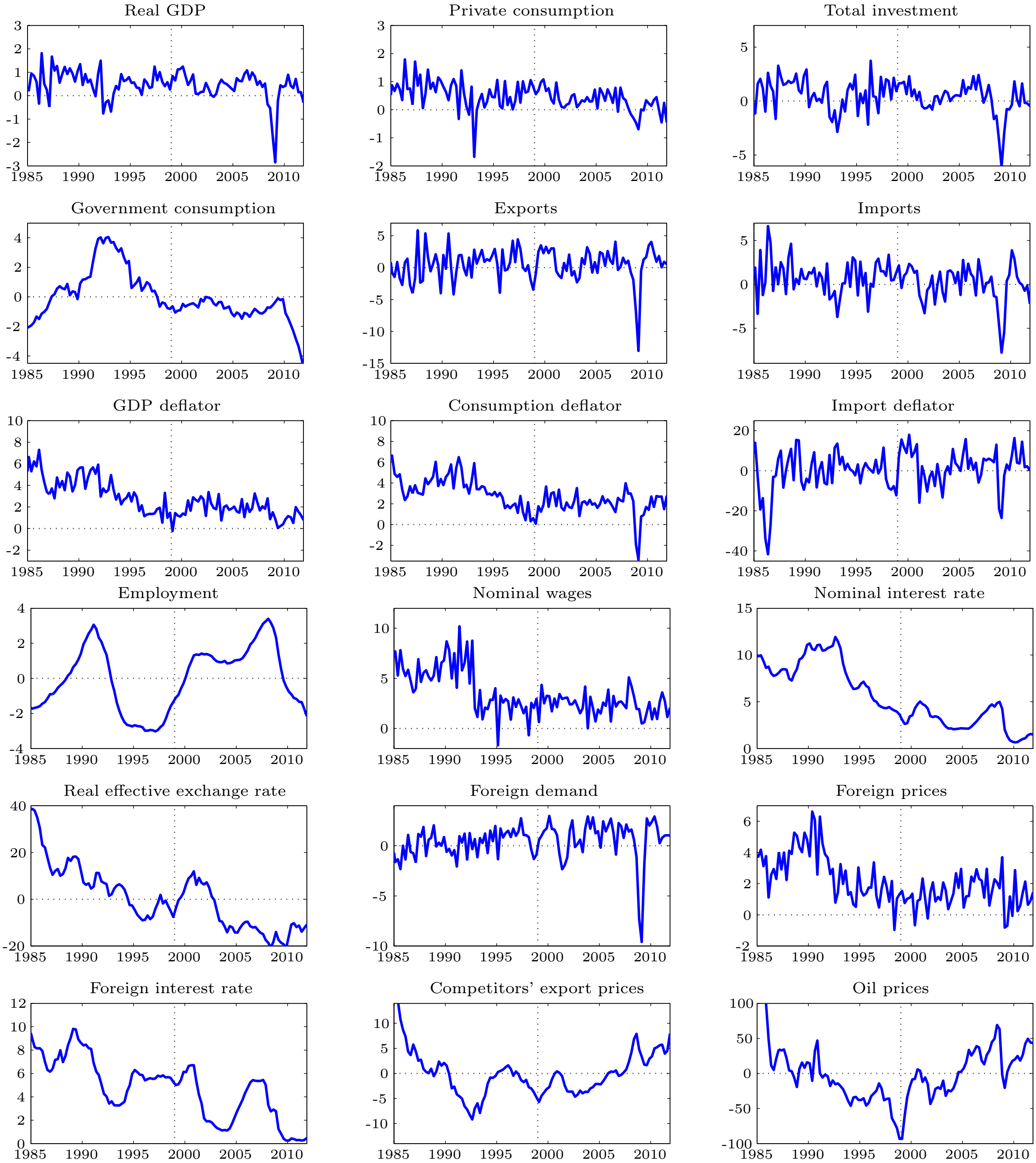
[4] We have used the standard weights $w(l, L) = 1 - l/(L+1)$, $l = 1, 2, \ldots, L$, for the autocovariances in the expression of the Newey and West (1987) estimator, where $L = 97$ when $N = 10000$. Our results are robust with respect to the choice of L with only minor fluctuations of the standard errors over the range $\{50, 51, \ldots, 500\}$. The standard error of the log predictive likelihood is computed with the delta method and is therefore based on the standard error of the predictive likelihood. The estimated autocovariances in the expression of the Newey and West estimator therefore rely on the conditional likelihood values of the N posterior draws. In our case, the N draws are not consecutive but taken 50 steps apart. The autocorrelation among the selected draws from the Markov chain is thereby considerably reduced compared with the case of consecutive draws. We have chosen to use a subset of the posterior draws for the NAWM and the DSGE-VAR since it significantly speeds up the calculations, without leading to substantial problems for the numerical precision. Our estimates are also robust to very small values for the conditional likelihood.

As suggested by one of the referees, across-chain based estimates is an alternative to the within-chain based numerical standard errors. That is, one computes the predictive likelihood estimates from multiple posterior samples and thereafter estimates the numerical standard errors across these samples. In Table E.3 we report evidence from one such exercise based on 20 Markov chains with parameter draws from the DSGE-VAR model when the historical sample ends in 2007Q4. Overall, the standard errors from the within chain estimates do not appear to underestimate the precision of the MC estimator.

In summary, we find that the numerical precision of the MC estimator of the log predictive likelihood is satisfactory. For very small values of the log predictive likelihood, the precision falls, especially for models with a large number of parameters, but it is unlikely that it will impair the validity of the ranking of models since such small values are rare. At the same time, the ranking is based on the log predictive score, i.e., the sum of the log predictive likelihood, and we have not attempted to assess the numerical accuracy of this measure. As a simple but rough rule of thumb for the large selection one may multiply the average standard error (approximated to be at most 0.1) by the number of forecasts (about 50), suggesting that models with a log predictive score differing by less than 5 log-units are approximately as good or as bad in this study. This strongly supports the view that our model ranking results, as documented in e.g. Figure 1 of the paper, are robust to estimation error.

Appendix E: Figures and Tables

Figure E.1: The euro area data for the sample 1985Q1–2011Q4.



Note: This figure shows the time series of the observable variables used in the estimation of the NAWM. Details on the variable transformations are provided in Christoffel, Coenen and Warne (2008, Section 3.2) or Section 2.3 in Christoffel, Coenen and Warne (2011). Price and wage inflation as well as the interest rates are reported in annualized percentage terms.

Figure E.2: Recursive estimates of the largest modulus of the BVAR when the parameters are evaluated at the posterior mean for the sample 1999Q1–2011Q4.

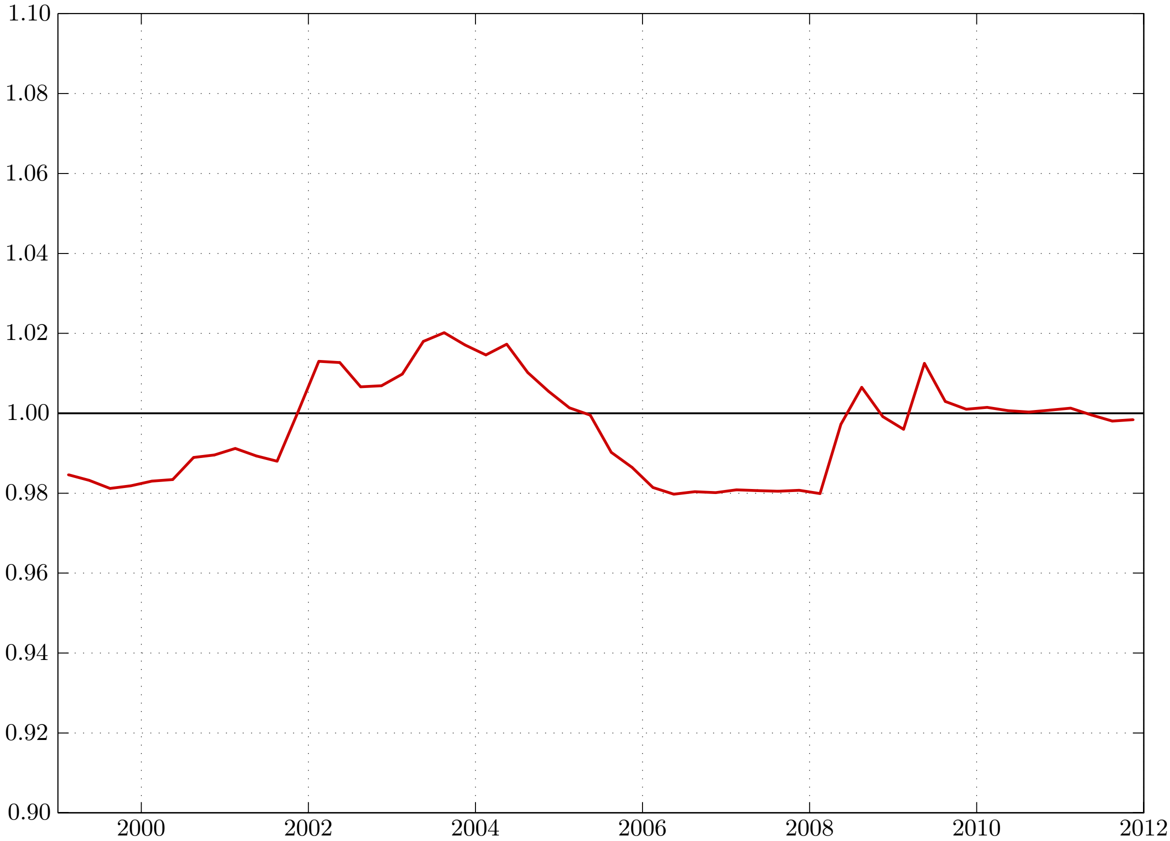


FIGURE E.3: Recursive posterior mean estimate of the constant term in the real GDP equation of the BVAR for the sample 1999Q1–2011Q4.

FIGURE E.4: Recursive forecast paths of real GDP growth and GDP deflator inflation using the mean of the marginalized predictive distributions for the sample 1999Q1–2011Q4.

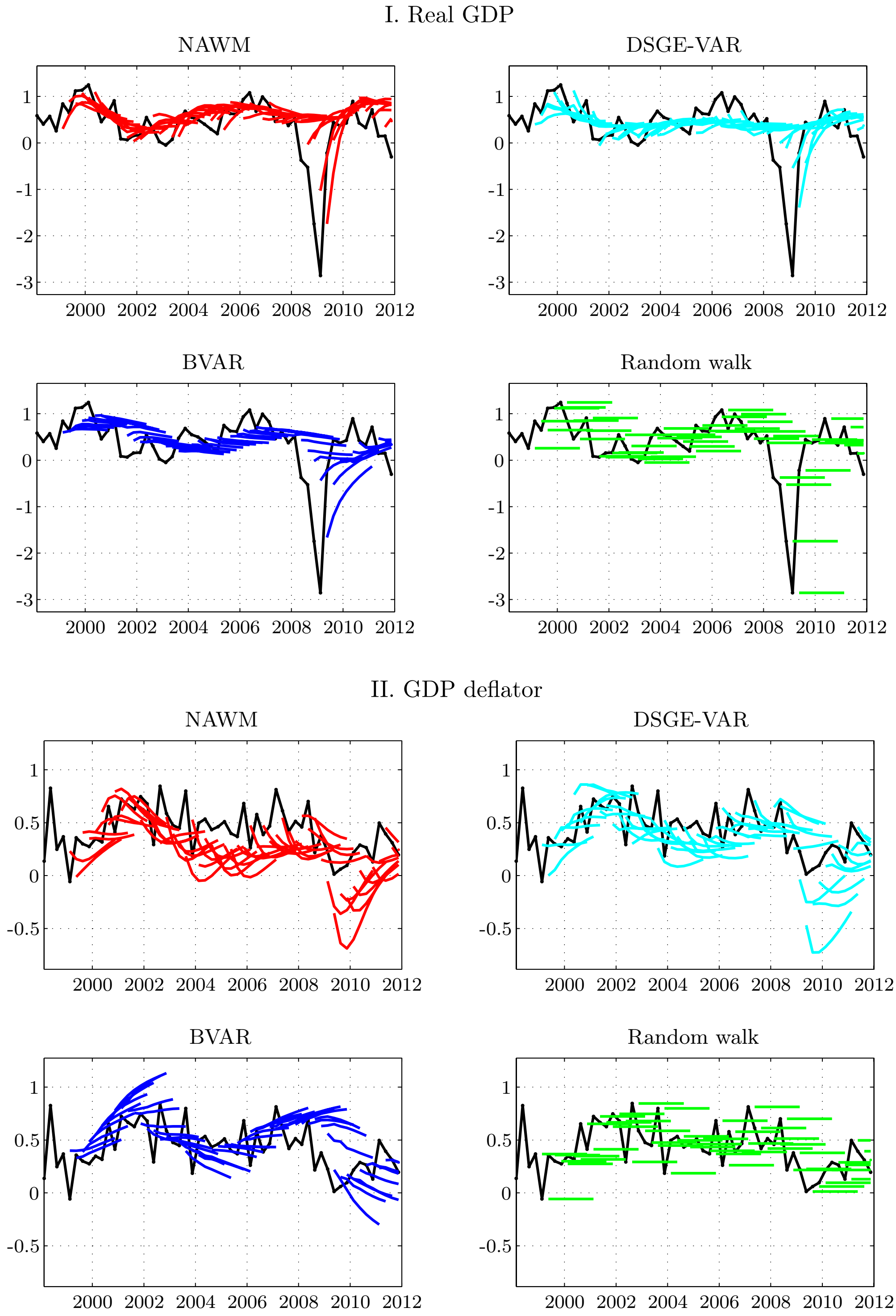


Figure E.5: Recursive forecast paths of the short-term nominal interest rate and private consumption growth using the mean of the marginalized predictive distributions for the sample 1999Q1–2011Q4.

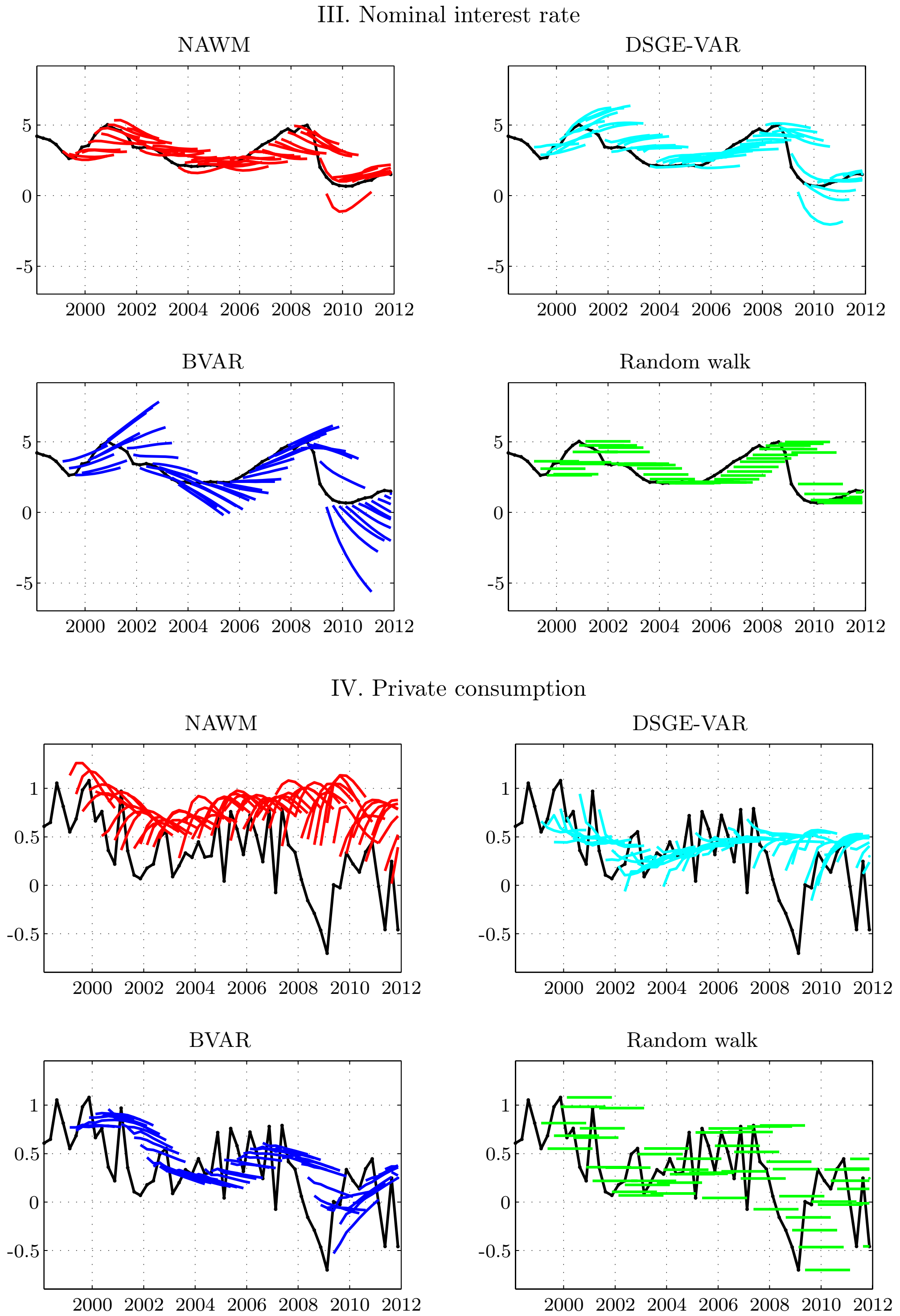


Figure E.6: Recursive forecast paths of total investment growth and employment using the mean of the marginalized predictive distributions for the sample 1999Q1–2011Q4.

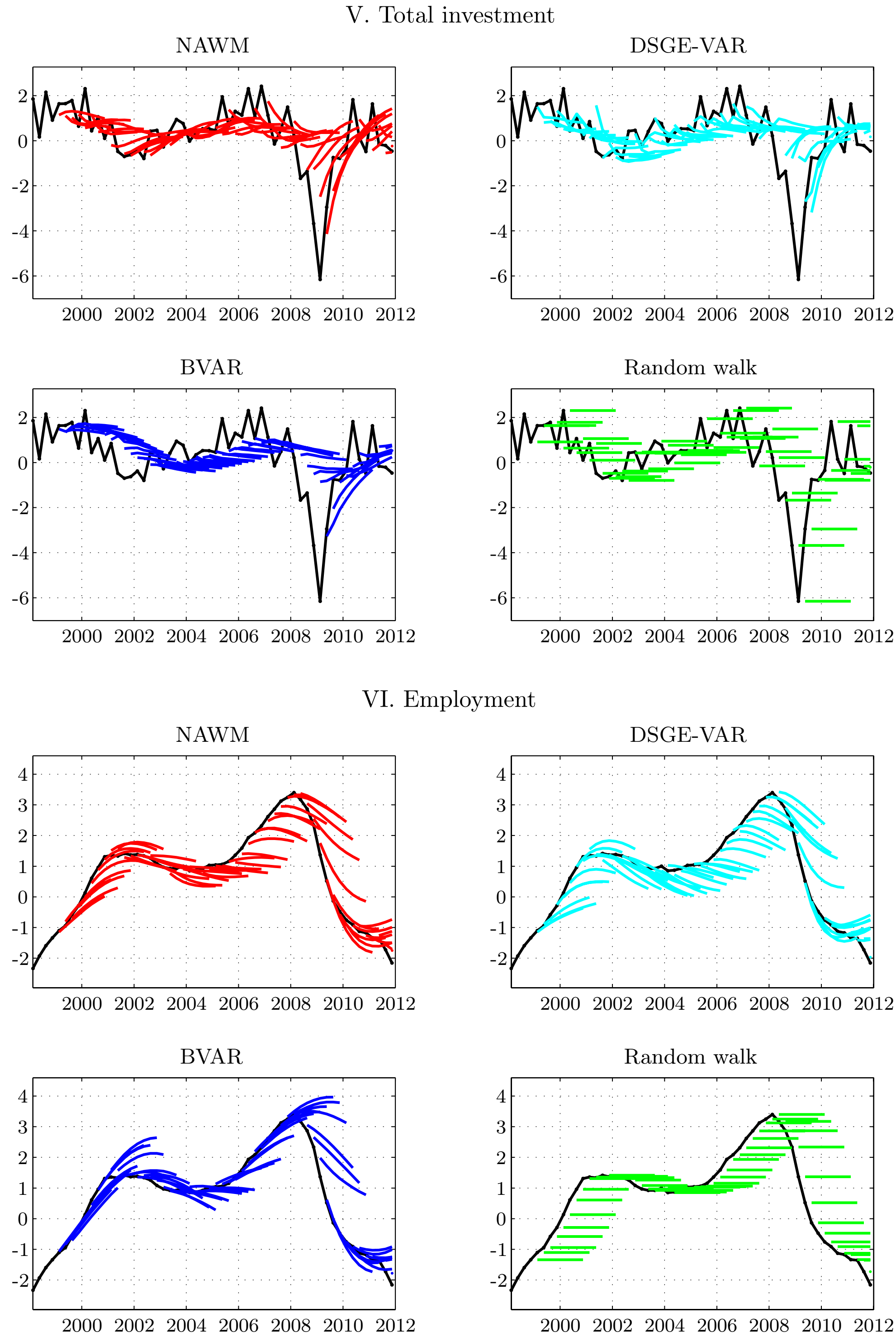


FIGURE E.7: Recursive forecast paths of nominal wage growth and import growth using the mean of the marginalized predictive distributions for the sample 1999Q1–2011Q4.

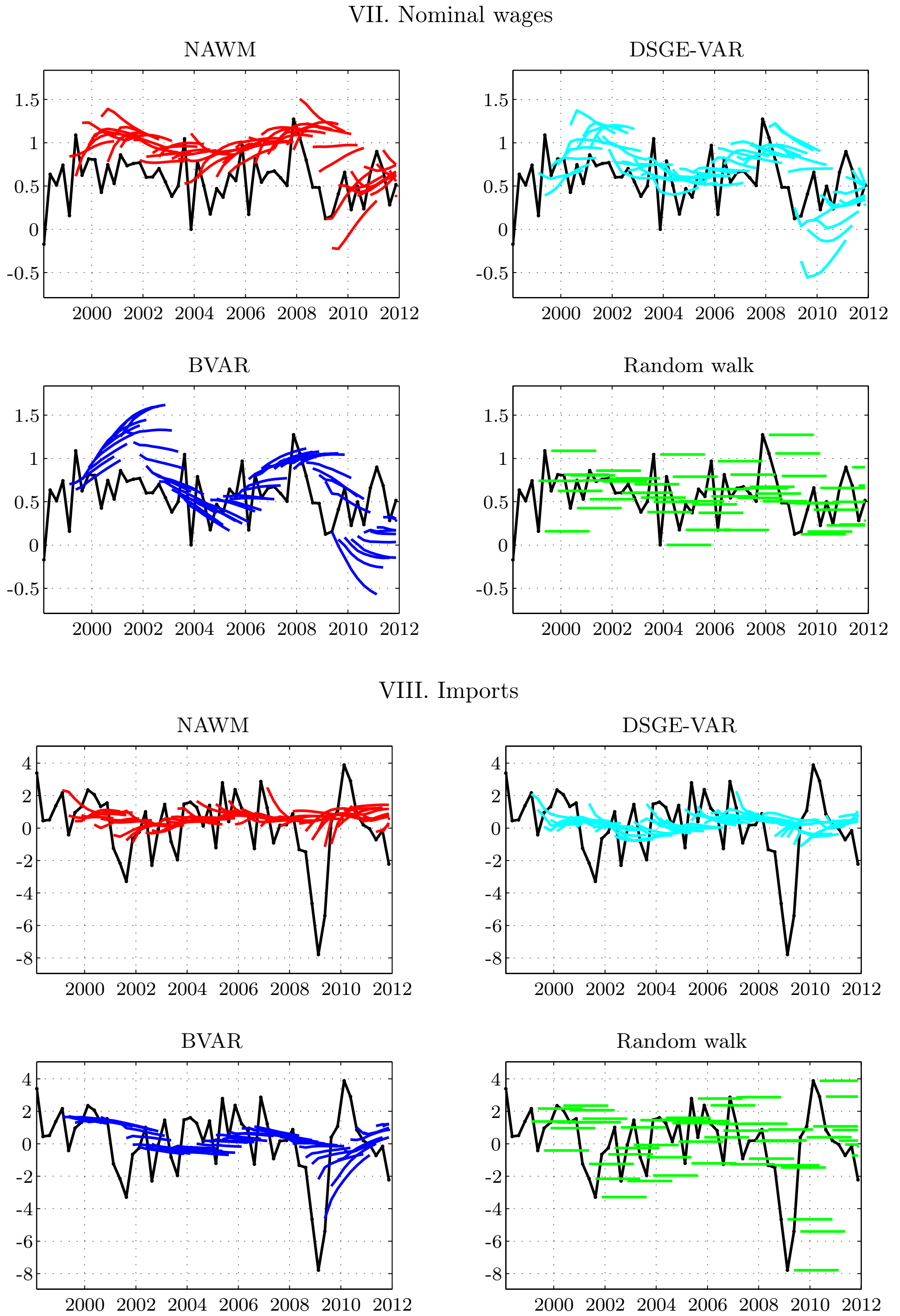


Figure E.8: Recursive forecast paths of export growth and consumption deflator inflation using the mean of the marginalized predictive distributions for the sample 1999Q1–2011Q4.

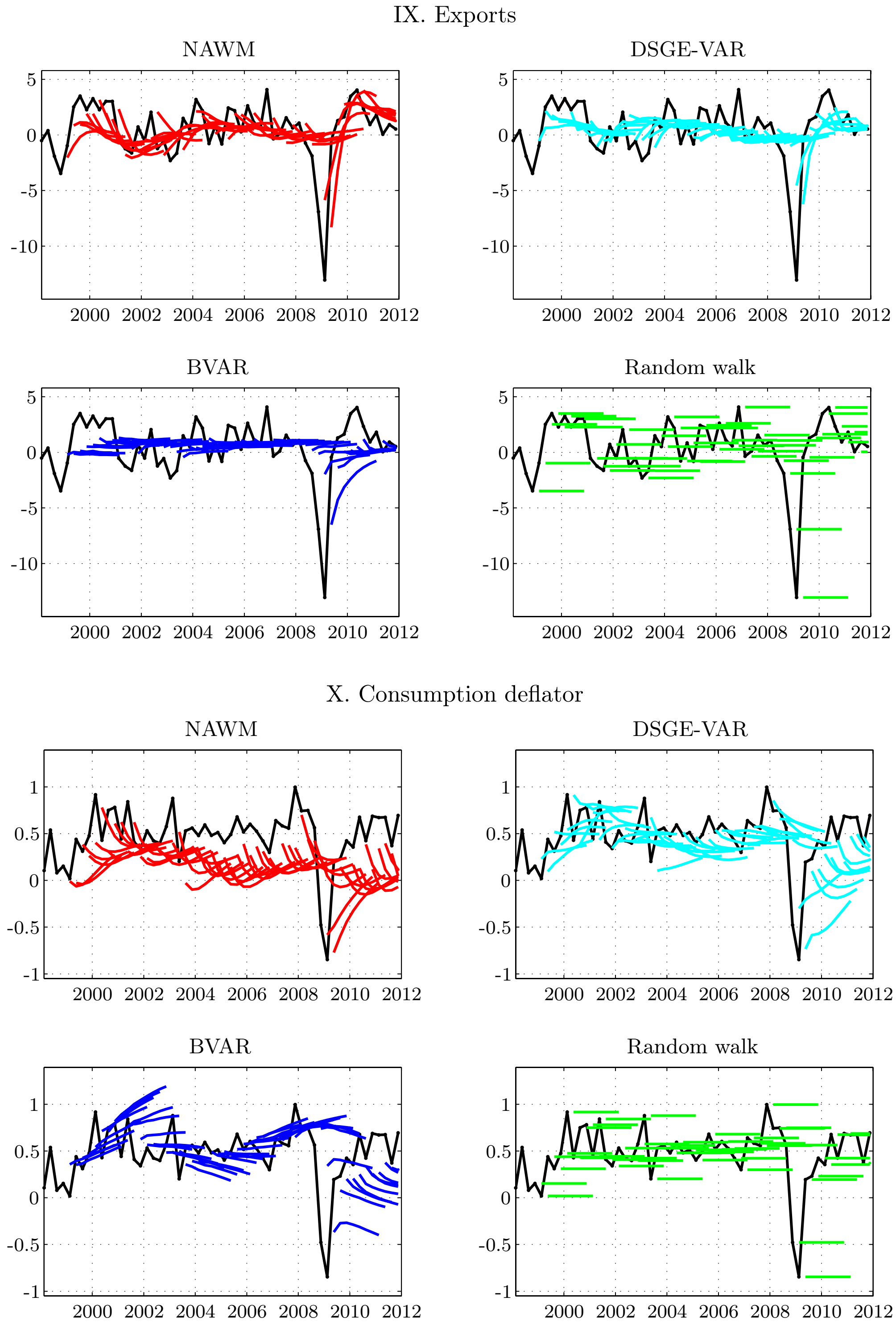


Figure E.9: Recursive forecast paths of import deflator inflation and the real effective exchange rate using the mean of the marginalized predictive distributions for the sample 1999Q1–2011Q4.

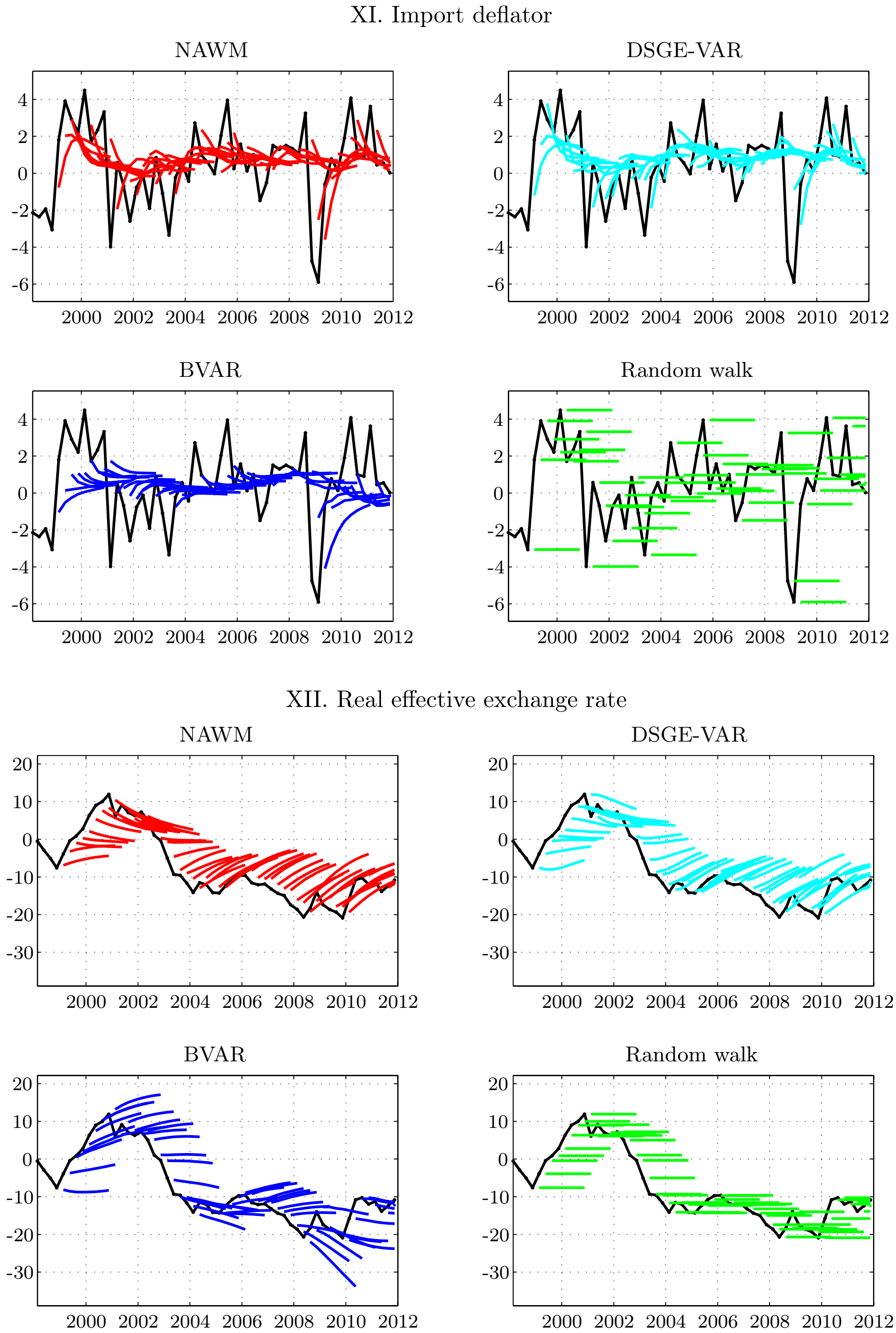


TABLE E.1: Differences in log predictive score between the MC estimator and the normal approximation for the large, medium, and small selections of variables over the sample 1999Q1–2011Q4.

	Large selection (12 variables)				Medium selection (7 variables)				Small selection (3 variables)			
Horizon	NAWM	DSGE-VAR	BVAR	RW	NAWM	DSGE-VAR	BVAR	RW	NAWM	DSGE-VAR	BVAR	RW
1	2.54	4.81	9.24	14.32	1.70	2.06	2.57	6.56	1.00	0.88	2.13	2.38
	(1.79)	(2.82)	(3.49)	(12.18)	(1.87)	(2.06)	(0.73)	(5.98)	(1.05)	(0.70)	(0.20)	(1.63)
2	3.28	10.62	14.29	19.82	2.28	4.30	7.41	9.26	1.61	2.07	5.35	3.29
	(1.44)	(4.38)	(2.33)	(15.93)	(2.25)	(3.29)	(2.06)	(8.19)	(1.44)	(1.10)	(0.02)	(2.34)
3	3.76	8.93	13.07	18.38	2.77	4.62	8.82	9.20	1.84	2.09	7.59	2.80
	(1.43)	(4.58)	(2.74)	(16.18)	(2.56)	(3.81)	(3.02)	(8.67)	(1.57)	(1.33)	(0.52)	(2.45)
4	3.99	8.85	12.59	15.88	3.19	4.81	10.35	8.88	1.90	1.99	6.44	2.32
	(1.58)	(4.47)	(2.12)	(14.70)	(2.86)	(4.04)	(2.76)	(8.47)	(1.55)	(1.36)	(0.36)	(2.16)
5	4.53	8.02	18.62	13.61	3.47	4.81	10.96	8.34	1.83	1.97	6.00	1.82
	(2.21)	(4.10)	(1.70)	(13.10)	(3.29)	(3.88)	(2.79)	(8.25)	(1.57)	(1.26)	(−0.19)	(1.47)
6	5.41	8.99	12.32	11.85	3.51	4.25	10.15	7.93	1.74	1.77	6.72	1.48
	(2.84)	(4.18)	(1.46)	(11.76)	(3.32)	(3.69)	(2.64)	(7.90)	(1.46)	(1.21)	(−0.04)	(1.47)
7	5.36	8.99	11.04	10.39	3.54	4.12	10.50	7.14	1.69	1.75	8.02	1.13
	(2.80)	(3.89)	(0.68)	(10.37)	(3.36)	(3.50)	(2.62)	(7.20)	(1.41)	(1.15)	(−0.39)	(1.18)
8	5.54	8.94	10.69	9.76	3.50	3.90	8.58	6.67	1.66	1.78	6.60	0.94
	(2.98)	(3.53)	(−0.64)	(9.70)	(3.30)	(3.26)	(0.75)	(6.67)	(1.36)	(1.13)	(−0.77)	(0.98)

Notes: The results within parentheses are based on dropping the density forecasts of the quarters 2008Q4 and 2009Q1 from the calculations. The log predictive likelihood for the random walk model is calculated with its analytical expression; see the online appendix (Appendix A) for details. For the NAWM and the DSGE-VAR models, 10,000 posterior draws have been taken from the available 500,000 post burn-in draws obtained via the random walk Metropolis sampler. The draws have been selected as draw number $1, 51, 101, \ldots, 499951$. For the BVAR direct sampling is possible and 50,000 posterior draws have been used; see the Appendix B.

TABLE E.2: Log predictive likelihood values with numerical standard errors based on the MC estimator for density forecasts over the forecast horizon 2008Q1–2009Q4 using historical data until 2007Q4.

	Large selection (12 variables)			Medium selection (7 variables)			Small selection (3 variables)		
Period	NAWM	DSGE-VAR	BVAR	NAWM	DSGE-VAR	BVAR	NAWM	DSGE-VAR	BVAR
2008Q1	−10.3279	−8.5661	−6.1484	−3.6295	−2.4129	−0.6359	−1.1646	−1.1503	−0.5581
	(0.0078)	(0.0127)	(0.0070)	(0.0074)	(0.0102)	(0.0050)	(0.0037)	(0.0070)	(0.0031)
2008Q2	−12.4433	−10.9317	−7.8271	−4.8907	−3.7637	−2.1863	−2.1381	−1.4539	−2.2022
	(0.0075)	(0.0123)	(0.0079)	(0.0067)	(0.0099)	(0.0064)	(0.0041)	(0.0053)	(0.0048)
2008Q3	−16.7118	−13.5161	−14.7368	−7.0085	−5.6422	−7.8644	−3.5873	−3.0935	−6.6163
	(0.0100)	(0.0125)	(0.0319)	(0.0084)	(0.0113)	(0.0240)	(0.0053)	(0.0097)	(0.0136)
2008Q4	−28.7516	−29.8833	−39.3785	−9.4763	−9.5283	−17.3812	−6.1870	−6.2509	−14.1912
	(0.0443)	(0.1210)	(0.8129)	(0.0162)	(0.0220)	(0.1335)	(0.0141)	(0.0179)	(0.0866)
2009Q1	−40.0379	−44.0610	−61.1553	−13.8284	−17.8481	−32.9921	−10.0689	−12.9329	−28.1760
	(0.0928)	(0.2685)	(0.9934)	(0.0264)	(0.1582)	(0.6053)	(0.0252)	(0.0951)	(0.2400)
2009Q2	−19.6019	−19.7989	−30.3377	−9.2210	−9.8641	−19.3259	−3.7704	−4.9328	−9.4277
	(0.0207)	(0.0382)	(0.4501)	(0.0191)	(0.0249)	(0.2059)	(0.0091)	(0.0169)	(0.0525)
2009Q3	−19.4668	−18.7053	−25.5558	−10.2396	−10.1233	−18.1275	−3.8423	−4.8870	−8.2950
	(0.0214)	(0.0295)	(0.3619)	(0.0220)	(0.0240)	(0.1948)	(0.0093)	(0.0154)	(0.0377)
2009Q4	−19.4219	−19.7333	−24.4852	−9.1823	−9.6789	−17.4570	−3.8024	−4.7036	−7.1406
	(0.0173)	(0.0401)	(0.3509)	(0.0183)	(0.0218)	(0.2381)	(0.0091)	(0.0148)	(0.0302)

Notes: Numerical standard errors are reported within parentheses. The standard errors of the NAWM and the DSGE-VAR have been computed with the Newey and West (1987) estimator. For these models, 10,000 posterior draws have been taken from the available 500,000 post burn-in draws obtained via the random walk Metropolis sampler. The draws have been selected as draw number $1, 51, 101, \ldots, 499951$. For the BVAR direct sampling is possible and 50,000 posterior draws have been computed; see Appendix B.

Table E.3: Standard deviations of the log predictive likelihood across 20 Markov chains and average within chain standard errors over the forecast period 2008Q1–2009Q4 using the DSGE-VAR model using the large selection.

1	2	3	4	5	6	7	8
Across chain standard deviations							
0.011117	0.010805	0.012263	0.111376	0.478660	0.029197	0.022633	0.021318
Average within chain standard errors							
0.012563	0.012347	0.013997	0.122407	0.328139	0.042445	0.032387	0.037675

Table E.4: RMSE for real GDP growth, GDP deflator inflation, and the short-term nominal interest rate in the small selection over the sample 1999Q1–2011Q4.

	Real GDP							
Model	1	2	3	4	5	6	7	8
NAWM	0.520*	0.658	0.678	0.707	0.719	0.726	0.736	0.736
DSGE-VAR	0.547	0.636*	0.640*	0.649*	0.652*	0.658*	0.670*	0.681*
BVAR	0.581	0.672	0.698	0.729	0.756	0.756	0.765	0.775
Random walk	0.542	0.750	0.852	0.942	1.000	1.018	1.027	1.089
	GDP deflator							
NAWM	0.250	0.287	0.332	0.336	0.316	0.304	0.274*	0.252*
DSGE-VAR	0.234	0.229	0.264	0.273	0.279*	0.287*	0.287	0.276
BVAR	0.197*	0.199*	0.233*	0.260*	0.285	0.315	0.340	0.367
Random walk	0.241	0.216	0.218	0.244	0.270	0.272	0.301	0.309
	Nominal interest rate							
NAWM	0.355	0.604*	0.791*	0.989*	1.162*	1.282*	1.359*	1.398*
DSGE-VAR	0.352*	0.643	0.902	1.166	1.423	1.632	1.816	1.954
BVAR	0.384	0.748	1.098	1.450	1.810	2.147	2.466	2.771
Random walk	0.452	0.798	1.079	1.318	1.537	1.715	1.851	1.962

Notes: An asterisk is used to indicate the smallest root-mean-squared error among the models for each forecast horizon.

TABLE E.5: RMSE for private consumption growth, total investment growth, employment, and nominal wage growth in the medium selection over the sample 1999Q1–2011Q4.

	Private consumption							
Model	1	2	3	4	5	6	7	8
NAWM	0.451	0.584	0.669	0.721	0.729	0.704	0.668	0.619
DSGE-VAR	0.376	0.382	0.407	0.424	0.422*	0.426*	0.434*	0.432*
BVAR	0.342*	0.350*	0.388*	0.411*	0.424	0.440	0.454	0.451
Random walk	0.385	0.376	0.404	0.475	0.458	0.477	0.528	0.502
	Total investment							
NAWM	1.083*	1.236*	1.315*	1.393*	1.454*	1.499*	1.521*	1.542*
DSGE-VAR	1.255	1.355	1.475	1.557	1.600	1.608	1.620	1.639
BVAR	1.224	1.402	1.484	1.581	1.680	1.687	1.721	1.767
Random walk	1.218	1.521	1.674	1.905	2.243	2.192	2.213	2.446
	Employment							
NAWM	0.148	0.304*	0.499*	0.679*	0.877*	1.058*	1.201*	1.328*
DSGE-VAR	0.144*	0.309	0.505	0.721	0.922	1.117	1.277	1.422
BVAR	0.158	0.329	0.513	0.706	0.907	1.111	1.300	1.480
Random walk	0.302	0.582	0.848	1.100	1.333	1.545	1.732	1.897
	Nominal wages							
NAWM	0.409	0.401	0.453	0.450	0.472	0.481	0.490	0.502
DSGE-VAR	0.355*	0.358	0.409	0.384	0.391	0.390	0.398	0.389*
BVAR	0.364	0.383	0.424	0.446	0.487	0.516	0.554	0.577
Random walk	0.381	0.336*	0.368*	0.381*	0.381*	0.368*	0.383*	0.410

Notes: An asterisk is used to indicate the smallest root-mean-squared error among the models for each forecast horizon.

Table E.6: RMSE for import growth, export growth, private consumption deflator inflation, import deflator inflation, and the real affective exchange rate in the large selection over the sample 1999Q1–2011Q4.

	Imports							
Model	1	2	3	4	5	6	7	8
NAWM	1.973	2.138*	2.163*	2.249*	2.271*	2.330	2.329	2.355
DSGE-VAR	2.089	2.197	2.207	2.292	2.317	2.322*	2.298*	2.323*
BVAR	1.903*	2.212	2.299	2.379	2.417	2.404	2.424	2.452
Random walk	2.041	2.619	3.107	3.275	3.503	3.327	3.293	3.180
	Exports							
NAWM	2.340	2.671	2.585*	2.559*	2.592*	2.662	2.766	2.779
DSGE-VAR	2.234*	2.613*	2.660	2.622	2.619	2.642*	2.713*	2.751*
BVAR	2.652	2.929	2.999	3.024	3.010	2.982	2.952	2.938
Random walk	2.768	3.560	3.951	4.383	4.383	4.395	4.248	4.279
	Consumption deflator							
NAWM	0.355	0.470	0.510	0.534	0.522	0.478	0.451	0.417
DSGE-VAR	0.313*	0.363*	0.398*	0.434*	0.437*	0.416*	0.410*	0.397*
BVAR	0.330	0.383	0.415	0.444	0.471	0.475	0.492	0.504
Random walk	0.320	0.389	0.418	0.478	0.490	0.465	0.479	0.474
	Import deflator							
NAWM	2.076	2.087*	2.055*	2.072*	2.041*	2.034*	2.027*	2.070*
DSGE-VAR	1.974*	2.130	2.086	2.095	2.114	2.102	2.114	2.149
BVAR	2.162	2.350	2.297	2.359	2.378	2.282	2.269	2.295
Random walk	2.573	2.996	2.768	3.224	3.441	3.294	3.336	3.294
	Real effective exchange rate							
NAWM	2.707	4.352	5.720	6.941	7.828	8.549	9.289	9.931
DSGE-VAR	2.618	4.077	5.355	6.536	7.424	8.208	9.083	9.832
BVAR	2.577	4.159	5.427	6.809	7.930	8.993	10.357	11.617
Random walk	2.486*	3.885*	4.982*	6.095*	6.956*	7.690*	8.556*	9.298*

Notes: An asterisk is used to indicate the smallest root-mean-squared error among the models for each forecast horizon.

TABLE E.7: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–1998Q4.

	Lag order p		
λ	2	3	4
0.9821	−1493.17		
1	−1482.45		
1.25	−1401.76		
1.3036		−1583.74	
1.5	−1366.42	−1489.76	
1.6250			−1670.21
2	−1334.22	−1392.88	−1505.70
2.5	−1321.46	−1354.54	−1419.24
3	−1315.96	−1335.77	−1377.70
3.5	−1313.66	−1325.37	−1354.11
4	−1312.91	−1320.34	−1340.82
4.5	−1312.90*	−1317.42	−1332.25
5	−1313.17	−1315.74	−1326.61
6	−1315.30	−1314.50	−1320.37
6.5	−1316.16	−1314.48*	−1318.74
7.5	−1317.81	−1315.00	−1317.08*
10	−1321.30	−1317.48	−1317.16
15	−1326.00	−1322.28	−1321.14
25	−1330.85	−1328.15	−1327.70
50	−1335.30	−1333.98	−1334.96
∞	−1340.25	−1341.12	−1343.19
NAWM	−1376.54		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal (Geweke, 1999, 2005) is −1376.10, while the same estimator yields −1321.07 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.8: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–1999Q4.

	Lag order p		
λ	2	3	4
0.9167	−1564.80		
1	−1521.27		
1.2167		−1658.23	
1.25	−1456.38	−1633.98	
1.5	−1425.90	−1531.62	
1.5167			−1748.02
2	−1398.13	−1449.68	−1547.33
2.5	−1387.41	−1416.29	−1473.89
3	−1382.95	−1400.55	−1437.88
3.5	−1381.65*	−1392.39	−1418.11
4	−1381.70	−1387.83	−1406.07
4.5	−1382.16	−1385.31	−1398.47
5	−1382.86	−1383.99	−1393.58
6	−1384.57	−1383.28*	−1388.36
6.5	−1385.46	−1383.44	−1387.08
7.5	−1387.17	−1384.25	−1385.95*
10	−1390.79	−1387.14	−1386.76
15	−1395.64	−1392.29	−1391.33
25	−1400.62	−1398.40	−1398.26
50	−1405.12	−1404.42	−1405.77
∞	−1410.27	−1412.09	−1414.32
NAWM	−1449.32		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1448.66, while the same estimator yields −1387.09 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.9: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2000Q4.

	Lag order p		
λ	2	3	4
0.8594	−1640.55		
1	−1573.18		
1.1406		−1735.26	
1.25	−1519.10	−1665.75	
1.4219			−1830.68
1.5	−1492.54	−1581.77	−1770.55
2	−1468.90	−1513.29	−1601.25
2.5	−1460.26	−1484.44	−1537.44
3	−1457.04	−1471.47	−1506.37
3.5	−1456.58*	−1464.85	−1488.91
4	−1457.20	−1461.38	−1478.47
4.5	−1458.08	−1459.66	−1472.02
5	−1459.10	−1458.95*	−1467.99
6	−1461.24	−1459.09	−1463.95
6.5	−1462.28	−1459.56	−1463.09
7.5	−1464.21	−1460.82	−1462.58*
10	−1468.14	−1464.36	−1464.26
15	−1473.23	−1469.99	−1469.48
25	−1478.32	−1476.30	−1476.71
50	−1482.88	−1482.17	−1848.23
∞	−1487.63	−1490.02	−1493.08
NAWM	−1530.58		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1530.17, while the same estimator yields −1459.98 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

Table E.10: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2001Q4.

	Lag order p		
λ	2	3	4
0.8088	−1721.46		
1	−1634.93		
1.0735		−1821.70	
1.25	−1588.25	−1717.04	
1.3382			−1917.96
1.5	−1565.13	−1647.45	−1805.70
2	−1545.05	−1585.70	−1662.77
2.5	−1538.31	−1560.42	−1606.35
3	−1536.38	−1549.13	−1579.10
3.5	−1536.17*	−1543.76	−1563.90
4	−1537.83	−1541.11	−1554.98
4.5	−1539.19	−1539.98	−1549.63
5	−1540.58	−1539.71*	−1546.43
6	−1543.27	−1540.48	−1543.57
6.5	−1544.51	−1541.18	−1543.14*
7.5	−1546.77	−1542.79	−1543.30
10	−1551.21	−1546.87	−1545.98
15	−1556.76	−1553.02	−1552.09
25	−1562.19	−1559.71	−1559.91
50	−1566.98	−1566.00	−1567.78
∞	−1572.24	−1574.23	−1577.73
NAWM	−1617.80		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1617.40, while the same estimator yields −1538.02 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.11: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2002Q4.

	Lag order p		
λ	2	3	4
0.7639	−1787.98		
1	−1685.59		
1.0139		−1893.04	
1.25	−1644.55	−1761.46	
1.2639			−1993.61
1.5	−1624.00	−1699.79	−1840.06
2	−1606.46	−1644.56	−1715.79
2.5	−1600.96	−1621.85	−1664.79
3	−1599.76*	−1612.00	−1640.04
3.5	−1599.94	−1607.44	−1626.25
4	−1602.05	−1605.34	−1618.23
4.5	−1603.68	−1604.60*	−1613.49
5	−1605.28	−1604.64	−1610.73
6	−1608.27	−1605.82	−1608.45
6.5	−1609.63	−1606.66	−1608.22*
7.5	−1612.06	−1608.51	−1608.67
10	−1616.77	−1612.91	−1611.76
15	−1622.59	−1619.34	−1618.20
25	−1628.21	−1626.21	−1626.20
50	−1633.12	−1641.25	−1634.18
∞	−1639.19	−1641.25	−1644.16
NAWM	−1686.97		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1686.59, while the same estimator yields −1600.77 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.12: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2003Q4.

	Lag order p		
λ	2	3	4
0.7237	−1862.64		
0.9605		−1970.82	
1	−1747.10	−1934.84	
1.1974			−2074.78
1.25	−1711.08	−1816.46	−2023.07
1.5	−1693.25	−1762.27	−1888.28
2	−1678.78	−1713.59	−1779.15
2.5	−1675.04	−1693.83	−1733.92
3	−1674.97*	−1686.28	−1712.22
3.5	−1676.54	−1682.90	−1700.31
4	−1678.74	−1681.70	−1693.61
4.5	−1680.78	−1681.65*	−1689.84
5	−1682.72	−1682.22	−1687.82
6	−1686.20	−1684.18	−1686.58*
6.5	−1687.74	−1685.32	−1686.73
7.5	−1690.46	−1687.62	−1687.74
10	−1695.61	−1692.71	−1691.62
15	−1701.85	−1699.72	−1698.68
25	−1708.01	−1706.86	−1706.95
50	−1713.37	−1713.87	−1714.72
∞	−1720.94	−1722.17	−1725.57
NAWM	−1769.15		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1768.80, while the same estimator yields −1674.97 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.13: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2004Q4.

	Lag order p		
λ	2	3	4
0.6875	−1925.45		
0.9125		−2037.71	
1	−1798.58	−1965.10	
1.1375			−2144.44
1.25	−1766.39	−1863.58	−2044.27
1.5	−1750.53	−1814.90	−1929.51
2	−1738.15	−1770.84	−1831.56
2.5	−1735.53*	−1753.07	−1790.59
3	−1736.16	−1746.74	−1770.93
3.5	−1737.49	−1744.09	−1760.29
4	−1740.72	−1743.40*	−1754.43
4.5	−1743.06	−1743.73	−1751.25
5	−1745.22	−1744.59	−1749.66
6	−1749.03	−1746.96	−1749.01*
6.5	−1750.70	−1748.24	−1749.36
7.5	−1753.63	−1750.76	−1750.66
10	−1759.19	−1756.15	−1759.95
15	−1765.87	−1763.22	−1762.26
25	−1772.12	−1771.22	−1770.00
50	−1777.42	−1778.27	−1779.44
∞	−1783.34	−1786.29	−1789.57
NAWM	−1836.41		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1836.18, while the same estimator yields −1735.34 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.14: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2005Q4.

	Lag order p		
λ	2	3	4
0.6548	−1984.63		
0.8690		−2100.71	
1	−1848.80	−1998.99	
1.0833			−2210.98
1.25	−1820.28	−1909.86	−2072.34
1.5	−1806.56	−1866.09	−1972.06
2	−1796.66	−1826.66	−1883.78
2.5	−1795.49*	−1811.22	−1846.78
3	−1797.12	−1806.29	−1829.32
3.5	−1799.36	−1804.64*	−1820.12
4	−1802.90	−1804.68	−1815.26
4.5	−1805.70	−1805.58	−1812.84
5	−1808.23	−1806.89	−1811.84*
6	−1812.61	−1809.94	−1812.02
6.5	−1814.50	−1811.47	−1812.69
7.5	−1817.78	−1814.38	−1815.48
10	−1823.84	−1820.26	−1819.52
15	−1830.94	−1828.83	−1827.14
25	−1837.48	−1837.05	−1837.16
50	−1843.00	−1844.31	−1845.90
∞	−1849.16	−1852.61	−1856.21
NAWM	−1906.02		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1905.71, while the same estimator yields −1795.30 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

Table E.15: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2006Q4.

	Lag order p		
λ	2	3	4
0.6250	−2046.93		
0.8295		−2164.11	
1	−1903.46	−2038.55	
1.0341			−2278.35
1.25	−1878.06	−1959.74	−2108.15
1.5	−1866.07	−1920.49	−2019.10
2	−1858.06	−1885.48	−1939.15
2.5	−1857.89*	−1872.38	−1905.40
3	−1860.16	−1868.48	−1890.08
3.5	−1863.09	−1867.69*	−1882.11
4	−1866.58	−1868.31	−1878.07
4.5	−1869.76	−1869.61	−1876.22
5	−1872.54	−1871.22	−1875.63*
6	−1877.24	−1874.66	−1876.38
6.5	−1879.23	−1876.33	−1877.22
7.5	−1882.67	−1879.40	−1879.29
10	−1888.94	−1885.94	−1884.61
15	−1896.22	−1894.69	−1893.28
25	−1902.89	−1902.74	−1902.69
50	−1908.50	−1909.77	−1911.22
∞	−1914.76	−1917.96	−1921.35
NAWM	−1975.10		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −1974.80, while the same estimator yields −1857.80 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

Table E.16: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2007Q4.

	Lag order p		
λ	2	3	4
0.5978	−2114.53		
0.7935		−2237.46	
0.9891			−2352.16
1	−1962.63	−2091.14	−2336.76
1.25	−1938.98	−2018.89	−2153.80
1.5	−1928.08	−1982.46	−2073.20
2	−1921.33*	−1949.82	−1999.54
2.5	−1922.04	−1937.55	−1968.33
3	−1924.80	−1934.33	−1954.23
3.5	−1927.99	−1933.76*	−1946.97
4	−1931.53	−1934.51	−1943.40
4.5	−1934.87	−1935.87	−1941.86
5	−1937.74	−1937.46	−1941.48*
6	−1942.56	−1940.84	−1942.44
6.5	−1944.60	−1942.96	−1943.32
7.5	−1948.10	−1946.47	−1945.06
10	−1954.50	−1953.11	−1951.64
15	−1961.91	−1961.32	−1960.20
25	−1968.72	−1969.29	−1969.26
50	−1974.44	−1976.26	−1977.67
∞	−1980.81	−1984.33	−1987.67
NAWM	−2047.30		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −2046.98, while the same estimator yields −1921.70 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.17: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2008Q4.

	Lag order p		
λ	2	3	4
0.5729	−2206.04		
0.7604		−2329.79	
0.9479			−2446.15
1	−2050.23	−2166.75	−2381.76
1.25	−2030.24	−2102.39	−2225.08
1.5	−2021.79	−2070.26	−2152.08
2	−2018.35*	−2042.67	−2085.74
2.5	−2021.34	−2033.42	−2058.55
3	−2025.70	−2032.34*	−2046.94
3.5	−2030.06	−2033.30	−2041.60
4	−2034.47	−2035.22	−2039.54
4.5	−2038.49	−2037.51	−2039.22*
5	−2041.91	−2039.86	−2039.84
6	−2047.56	−2044.37	−2042.39
6.5	−2049.92	−2046.73	−2043.90
7.5	−2053.93	−2050.96	−2046.91
10	−2061.13	−2058.71	−2054.72
15	−2069.27	−2067.89	−2064.90
25	−2076.59	−2076.54	−2075.11
50	−2082.64	−2083.94	−2084.25
∞	−2089.27	−2095.84	−2098.05
NAWM	−2157.16		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −2156.82, while the same estimator yields −2021.06 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

TABLE E.18: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2009Q4.

	Lag order p		
λ	2	3	4
0.5500	−2301.62		
0.7300		−2437.50	
0.9100			−2553.47
1	−2147.42	−2256.65	−2449.89
1.25	−2128.23	−2196.31	−2309.57
1.5	−2120.16	−2165.98	−2242.04
2	−2116.80*	−2139.84	−2179.60
2.5	−2119.26	−2131.34	−2153.28
3	−2123.57	−2130.01*	−2142.99
3.5	−2128.26	−2131.05	−2138.06
4	−2132.88	−2132.94	−2136.21
4.5	−2136.82	−2135.16	−2136.00*
5	−2140.21	−2137.38	−2136.68
6	−2145.80	−2142.33	−2139.23
6.5	−2148.14	−2144.63	−2140.71
7.5	−2152.12	−2148.63	−2143.84
10	−2159.27	−2156.22	−2151.71
15	−2167.38	−2165.33	−2161.83
25	−2174.68	−2173.96	−2172.04
50	−2180.70	−2181.35	−2181.16
∞	−2187.27	−2189.68	−2191.76
NAWM	−2256.87		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −2256.57, while the same estimator yields −2119.54 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

Table E.19: Estimates of the log marginal likelihood for the NAWM and the DSGE-VAR models over the sample 1985Q1–2010Q4.

	Lag order p		
λ	2	3	4
0.5288	−2386.58		
0.7019		−2514.81	
0.8750			−2635.27
1	−2214.49	−2318.49	−2498.25
1.25	−2196.91	−2262.48	−2370.82
1.5	−2189.78	−2234.34	−2307.86
2	−2187.50*	−2210.44	−2249.16
2.5	−2190.64	−2203.11	−2224.42
3	−2194.98	−2202.45*	−2214.93
3.5	−2199.92	−2203.97	−2210.55
4	−2204.38	−2206.22	−2209.07*
4.5	−2209.17	−2208.72	−2209.14
5	−2212.78	−2211.22	−2210.03
6	−2218.69	−2215.81	−2212.89
6.5	−2221.14	−2218.44	−2214.50
7.5	−2225.32	−2222.76	−2217.63
10	−2232.81	−2230.68	−2225.86
15	−2241.28	−2240.06	−2236.28
25	−2248.88	−2248.86	−2246.71
50	−2255.15	−2256.37	−2256.00
∞	−2262.01	−2268.87	−2266.76
NAWM	−2332.77		

Notes: An asterisk is used to indicate the largest log marginal likelihood estimate for each lag order using the Laplace approximation, where the Hessian matrix at the posterior mode has been computed with finite difference methods; see Abramowitz and Stegun (1964, p. 884) formulas 25.3.24 and 25.3.27. The estimated log marginal likelihood of the NAWM using the modified harmonic mean estimator based on a truncated normal is −2332.56, while the same estimator yields −2190.80 for the DSGE-VAR with $\lambda = 2.5$ and $p = 2$. The smallest value of λ which is consistent with a proper prior distribution for the DSGE-VAR is equal to $(n(p+1)+1)/T$; see Del Negro and Schorfheide (2004).

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