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Extending the NAWM with a partial indexation mechanism linking wages and trend productivity

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Abstract

This document sets out the details for extending the wage Phillips curve of the New Area-Wide Model (NAWM; cf. Christoffel, Coenen and Warne, 2008) with a partial indexation mechanism linking wages to trend productivity developments. The document first outlines the labour-market setting in which households are offering their labour services. It then derives the first-order condition characterising the optimal wage-setting decision of an individual household as well as the law of motion for the aggregate wage index. Finally, the document derives the implied log-linear wage Phillips curve. An appendix provides additional technical details of the derivations.

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Labour-Market Setup

Consider a continuum of households indexed by $h \in [0,1]$, each of which supplies differentiated labour services $N_{h,t}$ and acts as wage setter in monopolistically competitive markets. As a consequence, each household is committed to supply sufficient labour services to satisfy labour demand.

Further, there is a continuum of firms indexed by $f \in [0,1]$, all of which take the wage rates set by the households as given and aggregate the differentiated labour services into an homogenous bundle using a Dixit-Stiglitz technology,

$$N_{f,t} = \left(\int_0^1 \left(N_{f,t}^h \right)^{\frac{1}{\varphi_t}} dh \right)^{\varphi_t}, \tag{1}$$

where the possibly time-varying parameter $\varphi_t > 1$ is inversely related to the intratemporal elasticity of substitution between the differentiated labour services supplied by the households, $\eta_t = \varphi_t/(\varphi_t - 1) > 1$.[1]

Under these assumptions, each household h faces the following demand for its differentiated labour services from any given firm f as a function of its wage rate $W_{h,t}$ relative to the aggregate wage index W_t:

$$N_{f,t}^h = \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{\varphi_t}{\varphi_t - 1}} N_{f,t} \tag{2}$$

with $-\varphi_t/(\varphi_t - 1)$ representing the wage elasticity of labour demand.

Aggregating over the continuum of firms f, we obtain the following aggregate demand for the labour services of a given household h:

$$N_t^h = \int_0^1 N_{f,t}^h \, df = \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{\varphi_t}{\varphi_t - 1}} N_t, \tag{3}$$

which, in equilibrium, is met by the household's commitment to supply sufficient labour services for any wage rate set; that is,

$$N_{h,t} = N_t^h. \tag{4}$$

The wage index W_t can be obtained by substituting the labour index (1) into the labour demand schedule (2) and then integrating over the unit interval of households:

$$W_t = \left(\int_0^1 W_{h,t}^{\frac{1}{1-\varphi_t}} dh \right)^{1-\varphi_t}. \tag{5}$$

[1] As shown below, the parameter φ_t has a natural interpretation as a markup in the household-specific labour market.

The Optimal Wage-Setting Decision

Each household h supplies its differentiated labour services $N_{h,t}$ in monopolistically competitive markets. There is sluggish wage adjustment due to staggered wage contracts à la Calvo (1983). Accordingly, household h receives permission to optimally reset its nominal wage contract $W_{h,t}$ in a given period t with probability $1 - \xi$.

All households that receive permission to reset their wage contracts in a given period t choose the same wage rate $\tilde{W}_t = \tilde{W}_{h,t}$. Those households which do not receive permission are allowed to adjust their wage contracts according to the following scheme:

$$W_{h,t} \;=\; g^{\dagger}_{z,t}\, \Pi^{\dagger}_{C,t}\, W_{h,t-1}, \tag{6}$$

where $g^{\dagger}_{z,t} = g^{\tilde{\chi}}_{z,t}\, g^{1-\tilde{\chi}}_{z}$ and $\Pi^{\dagger}_{C,t} = \Pi^{\chi}_{C,t-1} \bar{\Pi}^{1-\chi}_{t}$. That is, the nominal wage contracts are indexed to a geometric average of the current (gross) rate of productivity growth, $g_{z,t} = z_t/z_{t-1}$, and the steady-state (gross) rate of productivity growth, g_z, and to a geometric average of past (gross) consumer price inflation, $\Pi_{C,t-1} = P_{C,t-1}/P_{C,t-2}$, and the monetary authority's possibly time-varying (gross) inflation objective, $\bar{\Pi}_t$, with $\tilde{\chi}$ and χ being indexation parameters.

Each household h receiving permission to reset its wage contract in period t maximises its lifetime utility function subject to its budget constraint, the demand for its differentiated labour services (3) and the wage-indexation scheme (6).

Hence, we obtain the following first-order condition characterising the households' optimal wage-setting decision:[2]

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi\beta)^k \left(\Lambda_{t+k}\, (1 - \tau^{N}_{t+k} - \tau^{W_h}_{t+k})\, g^{\dagger}_{z;t,t+k} \frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}} \frac{\tilde{W}_t}{P_{C,t}} - \varphi_{t+k}\, \epsilon^{N}_{t+k}\, (N_{h,t+k})^{\zeta} \right) N_{h,t+k} \right] \;=\; 0, \tag{7}$$

where Λ_{t+k} denotes the marginal utility out of income (equal across all individual households in the economy), $g^{\dagger}_{z;t,t+k} = g_z^{(1-\tilde{\chi})k} \prod_{s=1}^{k} g^{\tilde{\chi}}_{z,t+s}$, $\Pi^{\dagger}_{C;t,t+k} = \prod_{s=1}^{k} \Pi^{\chi}_{C,t+s-1} \bar{\Pi}^{1-\chi}_{t+s}$ and $\Pi_{C;t,t+k} = \prod_{s=1}^{k} \Pi_{C,t+s-1}$.

This expression states that in those labour markets in which wage contracts are re-optimised, the latter are set so as to equate the households' discounted sum of expected

[2] See the Appendix for details.

after-tax marginal revenues, expressed in consumption-based utility terms, λ_{t+k}, to the discounted sum of expected marginal cost, expressed in terms of marginal disutility of labour, $\Delta_{h,t+k} = -N_{h,t+k}^{\zeta}$. In the absence of wage staggering ($\xi = 0$), the factor φ_t represents a possibly time-varying markup of the real after-tax wage charged over the households' marginal rate of substitution between consumption and leisure,

$$(1 - \tau_t^N - \tau_t^{W_h}) \frac{\tilde{W}_t}{P_{C,t}} = - \varphi_t \, \epsilon_t^N \frac{\Delta_t}{\Lambda_t}, \tag{8}$$

reflecting the existence of monopoly power on the part of the households.[3]

The Aggregate Wage Index

With households setting the wage contracts for their differentiated labour services according to equation (6) and equation (7), respectively, the aggregate wage index W_t evolves according to

$$W_t = \left(\xi \left(g_{z,t} \, \Pi_{C,t}^{\dagger} W_{t-1} \right)^{\frac{1}{1-\varphi_t}} + (1 - \xi) \left(\tilde{W}_t \right)^{\frac{1}{1-\varphi_t}} \right)^{1-\varphi_t}. \tag{9}$$

The Log-Linear Wage Phillips Curve

We use $\widehat{\pi}_{C,t} = \log(\Pi_{C,t}/\bar{\Pi})$ to denote the logarithmic deviation of the current consumer price inflation rate from the monetary authority's long-run inflation objective, while $\widehat{\bar{\pi}}_t = \log(\bar{\Pi}_t/\bar{\Pi})$ represents the logarithmic deviation of the current possibly time-varying inflation objective from its long-run value. Moreover, because those firms which do not receive permission to reset their prices are allowed to index them to a geometric average of past inflation and the current inflation objective, it is natural to define the "quasi inflation gap" $\widehat{\dot{\pi}}_{C,t} = \widehat{\pi}_{C,t} - \widehat{\pi}_t^{\dagger}$, where $\widehat{\pi}_{C,t}^{\dagger} = \log(\Pi_{C,t}^{\dagger}/\bar{\Pi}) = \log(\Pi_{C,t-1}^{\chi} \bar{\Pi}_t^{1-\chi}/\bar{\Pi})$. Similarly, we define the "quasi productivity gap" $\widehat{\dot{g}}_{z,t} = \widehat{g}_{z,t} - \widehat{g}_{z,t}^{\dagger}$, where $\widehat{g}_{z,t}^{\dagger} = \log(g_{z,t}^{\dagger}/g_z) = \log(g_{z,t}^{\tilde{\chi}}/g_z^{\tilde{\chi}})$.

Finally, we use $\widehat{w}_t$, $\widehat{mrs}_t$ and $\widehat{\varphi}_t$ to denote, respectively, the productivity-adjusted real wage (deflated by the consumer price index and equal across all households), the marginal rate of substitution between consumption and leisure and the wage markup (all variables expressed as logarithmic deviations from their respective steady-state values).

[3] Note that, in this case, also the marginal disutility is equal across households; that is $\Delta_t = \Delta_{h,t}$.

Then, combining the log-linearised first-order condition characterising the households' optimal wage-setting decision (7) and the log-linearised aggregate wage index (5) yields the log-linear wage Phillips curve[4]

$$\begin{aligned}\widehat{w}_t \;&=\; \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{w}_{t+1}\right] + \frac{1}{1+\beta}\,\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{\widehat{\pi}}_{C,t+1}\right] - \frac{1}{1+\beta}\,\widehat{\widehat{\pi}}_{C,t} \\ &\quad + \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{\widehat{g}}_{z,t+1}\right] - \frac{1}{1+\beta}\,\widehat{\widehat{g}}_{z,t} - \frac{(1-\beta\xi)\,(1-\xi)}{(1+\beta)\,\xi\,\Psi(\varphi,\zeta)}\,\left(\widehat{w}_t^{\tau} - \widehat{mrs}_t - \widehat{\varphi}_t\right)\end{aligned} \tag{10}$$

with

$$\Psi(\varphi,\zeta) \;=\; 1 + \frac{\varphi}{\varphi-1}\,\zeta$$

and where $\widehat{w}_t^{\tau}$ represents the productivity-adjusted after-tax real wage (expressed in logarithmic deviation from its steady-state value) or, more compactly,

$$\begin{aligned}\Delta\,\widehat{w}_t \;&=\; \beta\,\mathrm{E}_t\left[\Delta\,\widehat{w}_{t+1}\right] + \beta\,\mathrm{E}_t\left[\widehat{\widehat{\pi}}_{C,t+1}\right] - \widehat{\widehat{\pi}}_{C,t} \\ &\quad + \beta\,\mathrm{E}_t\left[\widehat{\widehat{g}}_{z,t+1}\right] - \widehat{\widehat{g}}_{z,t} - \frac{(1-\beta\xi)\,(1-\xi)}{\xi\,\Psi(\varphi,\zeta)}\,\left(\widehat{w}_t^{\tau} - \widehat{mrs}_t - \widehat{\varphi}_t\right),\end{aligned} \tag{11}$$

which shows that it relates changes in the productivity-adjusted real wage to current and future deviations of the productivity-adjusted after-tax real wage from the markup over the marginal rate of substitution, accounting for developments in inflation and productivity.

Alternatively, noting that $\widehat{\widehat{\pi}}_{C,t} = \widehat{\pi}_{C,t} - \widehat{\pi}_{C,t}^{\dagger} = \widehat{\pi}_{C,t} - \chi\,\widehat{\pi}_{C,t-1} - (1-\chi)\,\widehat{\overline{\pi}}_t$ and $\widehat{\widehat{g}}_{z,t} = \widehat{g}_{z,t} - \widehat{g}_{z,t}^{\dagger} = (1-\tilde{\chi})\,\widehat{g}_{z,t}$, the wage Phillips curve can be written in terms of the productivity-adjusted (after-tax) real wage, actual consumer price inflation, the inflation objective and productivity developments as

$$\begin{aligned}\widehat{w}_t \;&=\; \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{w}_{t+1}\right] + \frac{1}{1+\beta}\,\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{\pi}_{C,t+1}\right] \\ &\quad - \frac{1+\beta\chi}{1+\beta}\,\widehat{\pi}_{C,t} + \frac{\chi}{1+\beta}\,\widehat{\pi}_{C,t-1} - \frac{\beta(1-\chi)}{1+\beta}\,\mathrm{E}_t\left[\widehat{\overline{\pi}}_{C,t+1}\right] + \frac{1-\chi}{1+\beta}\,\widehat{\overline{\pi}}_{C,t} \\ &\quad + \frac{\beta(1-\tilde{\chi})}{1+\beta}\,\mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] - \frac{1-\tilde{\chi}}{1+\beta}\,\widehat{g}_{z,t} - \frac{(1-\beta\xi)\,(1-\xi)}{(1+\beta)\,\xi\,\Psi(\varphi,\zeta)}\,\left(\widehat{w}_t^{\tau} - \widehat{mrs}_t - \widehat{\varphi}_t\right).\end{aligned} \tag{12}$$

Appendix

In this appendix, we provide details on the derivation of the first-order condition characterising the households' optimal wage-setting decision (7). We then log-linearise the latter

[4] For details see the Appendix.

as well as the aggregate wage index (9) around the deterministic steady state. We finally derive the log-linear wage Phillips curve (A.15).

The Optimal Wage-Setting Decision

To derive the first-order condition characterising the households' optimal wage-setting decision, we form the associated Lagrangean, substituting the labour demand schedule (3), the market-clearing condition (4), and the wage-indexation scheme (6), while neglecting all terms that are not relevant for the optimisation,

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi\beta)^k \left(-\frac{\epsilon^N_{t+k}}{1+\zeta} \left(\left(g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{W_{h,t}}{W_{t+k}} \right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}} N_{t+k} \right)^{1+\zeta} \right. \right. \tag{A.1}$$

$$\left. \left. + \Lambda_{t+k} \left(1 - \tau^N_{t+k} - \tau^{W_h}_{t+k}\right) g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{W_{h,t}}{P_{C,t+k}} \left(g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{W_{h,t}}{W_{t+k}} \right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}} N_{t+k} \right) \right]$$

or, more compactly,

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi\beta)^k \left(\Lambda_{t+k} \left(1 - \tau^N_{t+k} - \tau^{W_h}_{t+k}\right) \frac{W_{t+k}}{P_{C,t+k}} \left(g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{W_{h,t}}{W_{t+k}} \right)^{-\frac{1}{\varphi_{t+k}-1}} N_{t+k} \right. \right. \tag{A.2}$$

$$\left. \left. - \frac{\epsilon^N_{t+k}}{1+\zeta} \left(\left(g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{W_{h,t}}{W_{t+k}} \right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}} N_{t+k} \right)^{1+\zeta} \right) \right],$$

where Λ_{t+k} denotes the marginal utility out of income (equal across all households), $g^\dagger_{z;t,t+k} = g_z^{(1-\tilde{\chi})k} \prod_{s=1}^{k} g^{\tilde{\chi}}_{z,t+s}$, and $\Pi^\dagger_{C;t,t+k} = \prod_{s=1}^{k} \Pi^{\chi}_{C,t+s-1} \bar{\Pi}^{1-\chi}_{t+s}$.

Differentiating this expression with respect to $W_{h,t}$ then yields, after some algebra, the first-order condition characterising the households' optimal wage-setting decision,

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi\beta)^k \left(\Lambda_{t+k} \left(1 - \tau^N_{t+k} - \tau^{W_h}_{t+k}\right) g^\dagger_{z;t,t+k} \frac{\Pi^\dagger_{C;t,t+k}}{\Pi_{C;t,t+k}} \frac{\tilde{W}_t}{P_{C,t}} \right. \right. \tag{A.3}$$

$$\left. - \varphi_{t+k}\, \epsilon^N_{t+k} \left(\left(g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{\tilde{W}_t}{W_{t+k}} \right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}} N_{t+k} \right)^{\zeta} \right)$$

$$\left. \times \left(g^\dagger_{z;t,t+k}\, \Pi^\dagger_{C;t,t+k} \frac{\tilde{W}_t}{W_{t+k}} \right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}} N_{t+k} \right] = 0,$$

where $\tilde{W}_t$ denotes the optimal wage rate in period t that is chosen by all households that have received permission to reset their wage contracts and $\Pi_{C;t,t+k} = \prod_{s=1}^{k} \Pi_{C,t+s-1}$.

Invoking again the demand schedule (3) in combination with the market-clearing condition (4) and the indexation scheme (6), the first-order condition (A.3) simplifies to

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\xi\beta)^k\left(\Lambda_{t+k}\,(1-\tau^N_{t+k}-\tau^{W_h}_{t+k})\,g^{\dagger}_{z;t,t+k}\,\frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\,\frac{\tilde{W}_t}{P_{C,t}}\right.\right. \tag{A.4}$$

$$\left.\left.-\,\varphi_{t+k}\,\epsilon^N_{t+k}\,(N_{h,t+k})^{\zeta}\right)N_{h,t+k}\right]\;=\;0,$$

(cf. equation (7) in the main text).

Log-Linearisation of the Optimal Wage-Setting Decision

In order to log-linearise the households' optimal wage-setting decision, it is convenient to define the auxiliary variables $x_t=\tilde{W}_t/W_t$, $w_t=W_t/(z_tP_{C,t})$ and $\lambda_t=z_t\,\Lambda_t$. Substituting these variables, the first-order condition (A.3) can be re-written as

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\xi\beta)^k\left(\lambda_{t+k}\,(1-\tau^N_{t+k}-\tau^{W_h}_{t+k})\,w_{t+k}\left(\frac{g^{\dagger}_{z;t,t+k}}{g_{z;t,t+k}}\,\frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\,x_t\,\frac{w_t}{w_{t+k}}\right)^{-\frac{1}{\varphi_{t+k}-1}}N_{t+k}\right.\right. \tag{A.5}$$

$$\left.\left.-\,\varphi_{t+k}\,\epsilon^N_{t+k}\left(\left(\frac{g^{\dagger}_{z;t,t+k}}{g_{z;t,t+k}}\,\frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\,x_t\,\frac{w_t}{w_{t+k}}\right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}}N_{t+k}\right)^{1+\zeta}\right)\right]\;=\;0,$$

or, after re-arranging,

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\xi\beta)^k\,\lambda_{t+k}\,(1-\tau^N_{t+k}-\tau^{W_h}_{t+k})\,w_{t+k}\left(\frac{g^{\dagger}_{z;t,t+k}}{g_{z;t,t+k}}\,\frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\,x_t\,\frac{w_t}{w_{t+k}}\right)^{-\frac{1}{\varphi_{t+k}-1}}N_{t+k}\right] \tag{A.6}$$

$$=\;\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\xi\beta)^k\,\varphi_{t+k}\,\epsilon^N_{t+k}\left(\left(\frac{g^{\dagger}_{z;t,t+k}}{g_{z;t,t+k}}\,\frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\,x_t\,\frac{w_t}{w_{t+k}}\right)^{-\frac{\varphi_{t+k}}{\varphi_{t+k}-1}}N_{t+k}\right)^{1+\zeta}\right]$$

where $g_{z;t,t+k}=\prod_{s=1}^{k}g_{z,t+s-1}$.

Indicating the logarithmic deviation of a variable from its steady-state value by a hat ('$\widehat{\ }$'), defining steady-state values implicitly by dropping the time subscripts, and noting that $\Pi^{\dagger}_C=\Pi$, $x=\epsilon^N=1$, we obtain the following log-linearised expression:

$$\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\,\lambda\,(1-\tau^N-\tau^{W_h})\,w\,N\left(\widehat{\lambda}_{t+k}-\frac{\widehat{\tau}^N+\widehat{\tau}^{W_h}}{1-\tau^N-\tau^{W_h}}+\widehat{w}_{t+k}\right. \tag{A.7}$$

$$\left.-\,\frac{1}{\varphi-1}\left(\sum_{s=1}^{k}\left(\widehat{\pi}^{\dagger}_{C,t+s}-\widehat{\pi}_{C,t+s}+\widehat{g}^{\dagger}_{z,t+s}-\widehat{g}_{z,t+s}\right)+\widehat{x}_t+\widehat{w}_t-\widehat{w}_{t+k}\right)+\widehat{N}_{t+k}\right)\Bigg]$$

$$=\;\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\,\varphi\,N^{1+\zeta}\Bigg(\widehat{\varphi}_{t+k}+\widehat{\epsilon}^N_{t+k}-\frac{(1+\zeta)\,\varphi}{\varphi-1}\Bigg(\sum_{s=1}^{k}\Big(\widehat{\pi}^{\dagger}_{C,t+s}-\widehat{\pi}_{C,t+s}$$

$$+\widehat{g}^{\dagger}_{z,t+s} - \widehat{g}_{z,t+s}\Big) + \widehat{x}_t + \widehat{w}_t - \widehat{w}_{t+k}\Big) + (1+\zeta)\,\widehat{N}_{t+k}\Big)\Big]$$

or, noting that $(1-\tau^N-\tau^{W_h})\,w = \varphi\,N^{\zeta}/\lambda$ (cf. equation (8) in the main text) and rearranging,

$$\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\Bigg(\widehat{w}^{\tau}_{t+k} - \widehat{mrs}_{t+k} - \widehat{\varphi}_{t+k} + \Psi(\varphi,\zeta)\,\Bigg(\sum_{s=1}^{k}\Big(\widehat{\pi}^{\dagger}_{C,t+s} - \widehat{\pi}_{C,t+s} + \widehat{g}^{\dagger}_{z,t+s} - \widehat{g}_{z,t+s}\Big) + \widehat{x}_t + \widehat{w}_t - \widehat{w}_{t+k}\Bigg)\Bigg)\Bigg] = 0, \tag{A.8}$$

where the terms

$$\widehat{w}^{\tau}_{t+k} = -\frac{\widehat{\tau}^N + \widehat{\tau}^{W_h}}{1-\tau^N-\tau^{W_h}} + \widehat{w}_{t+k},$$

$$\widehat{mrs}_{t+k} = \widehat{\epsilon}^N_{t+k} + \zeta\,\widehat{N}_{t+k} - \widehat{\lambda}_{t+k}$$

denote the log-linearised productivity-adjusted after-tax real wage and the log-linearised marginal rate of substitution, respectively, and

$$\Psi(\varphi,\zeta) = 1 + \frac{\varphi}{\varphi-1}\,\zeta.$$

Solving for $\widehat{x}_t + \widehat{w}_t$, we obtain

$$\begin{aligned}
\widehat{x}_t + \widehat{w}_t &= -(1-\beta\xi)\,\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\Bigg(\Psi(\varphi,\zeta)^{-1}\,(\widehat{w}^{\tau}_{t+k} - \widehat{mrs}_{t+k} - \widehat{\varphi}_{t+k}) \\
&\qquad - \sum_{s=1}^{k}\Big(\widehat{\pi}_{C,t+s} - \widehat{\pi}^{\dagger}_{C,t+s} + \widehat{g}^{\dagger}_{z,t+s} - \widehat{g}_{z,t+s}\Big) - \widehat{w}_{t+k}\Bigg)\Bigg] \\
&= -(1-\beta\xi)\,\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\,\Big(\Psi(\varphi,\zeta)^{-1}\,(\widehat{w}^{\tau}_{t+k} - \widehat{mrs}_{t+k} - \widehat{\varphi}_{t+k}) - \widehat{w}_{t+k}\Big)\Bigg] \\
&\qquad + (1-\beta\xi)\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\sum_{s=0}^{k-1}\Big(\widehat{\pi}_{C,t+s+1} - \widehat{\pi}^{\dagger}_{C,t+s+1} + \widehat{g}^{\dagger}_{z,t+s} - \widehat{g}_{z,t+s}\Big)\Bigg] \\
&= -(1-\beta\xi)\,\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\,\Big(\Psi(\varphi,\zeta)^{-1}\,(\widehat{w}^{\tau}_{t+k} - \widehat{mrs}_{t+k} - \widehat{\varphi}_{t+k}) - \widehat{w}_{t+k}\Big)\Bigg] \\
&\qquad + \beta\xi\,\mathrm{E}_t\Bigg[\sum_{k=0}^{\infty}(\beta\xi)^k\,\Big(\widehat{\pi}_{C,t+k+1} - \widehat{\pi}^{\dagger}_{C,t+k+1} + \widehat{g}^{\dagger}_{z,t+s} - \widehat{g}_{z,t+s}\Big)\Bigg].
\end{aligned} \tag{A.9}$$

Forming the quasi-difference $Z_t - \beta\xi\,\mathrm{E}_t[Z_{t+1}]$ then yields

$$\begin{aligned}
\widehat{x}_t + \widehat{w}_t - \beta\xi\,\mathrm{E}_t\,[\widehat{x}_{t+1} + \widehat{w}_{t+1}] &= -(1-\beta\xi)\,\Psi(\varphi,\zeta)^{-1}\,(\widehat{w}^{\tau}_t - \widehat{mrs}_t - \widehat{\varphi}_t) \\
&\quad + (1-\beta\xi)\,\widehat{w}_t + \beta\xi\,\mathrm{E}_t\Big[\hat{\dot{\pi}}_{C,t+1}\Big] + \beta\xi\,\mathrm{E}_t\Big[\hat{\dot{g}}_{z,t+1}\Big],
\end{aligned} \tag{A.10}$$

where $\hat{\dot{\pi}}_{C,t} = \widehat{\pi}_{C,t} - \widehat{\pi}^{\dagger}_{C,t}$ and $\hat{\dot{g}}_{z,t} = \widehat{g}_{z,t} - \widehat{g}^{\dagger}_{z,t}$.

Log-Linearisation of the Aggregate Wage Index

To log-linearise the aggregate wage index (9), we first re-write the identity as

$$1 \;=\; \left(\xi \left(\frac{g^{\dagger}_{z,t}}{g_{z,t}} \frac{\Pi^{\dagger}_{C,t}}{\Pi_{C,t}} \frac{w_{t-1}}{w_t} \right)^{\frac{1}{1-\varphi_t}} + (1-\xi)\,(x_t)^{\frac{1}{1-\varphi_t}} \right)^{1-\varphi_t}. \tag{A.11}$$

Log-linearisation then yields

$$\widehat{x}_t \;=\; \frac{\xi}{1-\xi} \left(\widehat{w}_t - \widehat{w}_{t-1} + \hat{\dot{\pi}}_{C,t} + \hat{\dot{g}}_{z,t} \right). \tag{A.12}$$

The Log-Linear Wage Phillips Curve

Combining equations (A.10) and (A.12) yields the log-linear wage Phillips-curve

$$\begin{aligned} \widehat{w}_t \;=\;& \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{w}_{t+1}\right] + \frac{1}{1+\beta}\,\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\hat{\dot{\pi}}_{C,t+1}\right] - \frac{1}{1+\beta}\,\hat{\dot{\pi}}_{C,t} \\ &+ \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\hat{\dot{g}}_{z,t+1}\right] - \frac{1}{1+\beta}\,\hat{\dot{g}}_{z,t} - \frac{(1-\beta\xi)\,(1-\xi)}{(1+\beta)\,\xi\,\Psi(\varphi,\zeta)}\,\left(\widehat{w}^{\tau}_t - \widehat{mrs}_t - \widehat{\varphi}_t\right), \end{aligned} \tag{A.13}$$

or, more compactly,

$$\begin{aligned} \Delta\,\widehat{w}_t \;=\;& \beta\,\mathrm{E}_t\left[\Delta\,\widehat{w}_{t+1}\right] + \beta\,\mathrm{E}_t\left[\hat{\dot{\pi}}_{C,t+1}\right] - \hat{\dot{\pi}}_{C,t} \\ &+ \beta\,\mathrm{E}_t\left[\hat{\dot{g}}_{z,t+1}\right] - \hat{\dot{g}}_{z,t} - \frac{(1-\beta\xi)\,(1-\xi)}{\xi\,\Psi(\varphi,\zeta)}\,\left(\widehat{w}^{\tau}_t - \widehat{mrs}_t - \widehat{\varphi}_t\right), \end{aligned} \tag{A.14}$$

Alternatively, recalling that $\hat{\dot{\pi}}_{C,t} = \widehat{\pi}_{C,t} - \widehat{\pi}^{\dagger}_{C,t} = \widehat{\pi}_{C,t} - \chi\,\widehat{\pi}_{C,t-1} - (1-\chi)\,\hat{\bar{\pi}}_t$ and $\hat{\dot{g}}_{z,t} = \widehat{g}_{z,t} - \widehat{g}^{\dagger}_{z,t} = (1-\tilde{\chi})\,\widehat{g}_{z,t}$, the wage Phillips curve can be written in terms of the productivity-adjusted (after-tax) real wage, actual consumer price inflation, the inflation objective and productivity developments as

$$\begin{aligned} \widehat{w}_t \;=\;& \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{w}_{t+1}\right] + \frac{1}{1+\beta}\,\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{\pi}_{C,t+1}\right] \\ &- \frac{1+\beta\chi}{1+\beta}\,\widehat{\pi}_{C,t} + \frac{\chi}{1+\beta}\,\widehat{\pi}_{C,t-1} - \frac{\beta(1-\chi)}{1+\beta}\,\mathrm{E}_t\left[\hat{\bar{\pi}}_{C,t+1}\right] + \frac{1-\chi}{1+\beta}\,\hat{\bar{\pi}}_{C,t} \\ &+ \frac{\beta(1-\tilde{\chi})}{1+\beta}\,\mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] - \frac{1-\tilde{\chi}}{1+\beta}\,\widehat{g}_{z,t} - \frac{(1-\beta\xi)\,(1-\xi)}{(1+\beta)\,\xi\,\Psi(\varphi,\zeta)}\,\left(\widehat{w}^{\tau}_t - \widehat{mrs}_t - \widehat{\varphi}_t\right) \end{aligned} \tag{A.15}$$

(cf. equation (A.15) in the main text).

References

Christoffel, Kai, Günter Coenen, and Anders Warne, 2008, "The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis", European Central Bank Working Paper No. 944.