Identifying the Influences
of Nominal and Real Rigidities
in Aggregate Price-Setting Behavior

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Abstract

We formulate a generalized price-setting framework that incorporates staggered contracts of multiple durations and that enables us to directly identify the influences of nominal vs. real rigidities. We estimate this framework using macroeconomic data for Germany (1975-98) and for the United States (1983-2003). In each case, we find that the data is well-characterized by nominal contracts with an average duration of about two to three quarters. We also find that new contracts exhibit very low sensitivity to marginal cost, corresponding to a relatively high degree of real rigidity. Finally, our results indicate that backward-looking price-setting behavior (such as indexation to lagged inflation) is not needed in explaining the aggregate data, at least in an environment with a stable monetary policy regime and a transparent and credible inflation objective.
1 Introduction

Micro-founded models of price-setting behavior are essential for understanding aggregate inflation dynamics and for evaluating the performance of monetary policy regimes. Both nominal and real rigidities play a crucial role in determining the particular implications of these models; thus, a large body of empirical research has been oriented towards gauging the frequency of price adjustment, the sensitivity of price revisions to demand and cost pressures, and the prevalence of indexation or rules of thumb.

The recent empirical literature has mainly focused on estimating variants of the New Keynesian Phillips Curve (NKPC), which can be derived under the Calvo (1983) assumption that price contracts have random duration with a constant hazard rate. Nevertheless, since the slope of the NKPC depends on the mean duration of price contracts as well as potential sources of real rigidity, the underlying structural parameters cannot be separately identified using this framework. Furthermore, while most studies have obtained highly significant estimates of the coefficient on lagged inflation, no consensus has been reached about whether to interpret these results as reflecting backward-looking price-setting behavior or gradual learning about occasional shifts in the monetary policy regime.

In this paper, we formulate a generalized price-setting framework that incorporates staggered contracts of multiple durations and that enables us to directly identify the influences of nominal vs. real rigidities. In analyzing contracts with random duration, we assume that every firm which resets its price faces the same ex ante probability distribution of contract duration, but we do not impose any restrictions on the shape of the hazard function. In analyzing contracts with fixed duration, we assume that each firm signs price contracts

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2 The importance of combining nominal and real rigidities has been emphasized by Ball and Romer (1990), Chari, Kehoe, and McGratten (2000), and Christiano, Eichenbaum, and Evans (2005).

3 Following Gali and Gertler (1999) and Sbordone (2002), the literature has become too voluminous to be enumerated here; recent examples include Lindé (2001), and Cogley and Sbordone (2004).


5 For example, Galí and Gertler (1999) consider a specification with rule-of-thumb price-setters, while Erceg and Levin (2003) show that the lagged inflation term in the hybrid Phillips curve can be generated by rational agents who use signal extraction to learn about shifts in the central bank’s inflation objective.

6 Mash (2003) uses micro evidence to calibrate a similar price-setting framework with a generalized hazard function, and shows that the calibrated model can roughly match empirical autocorrelations.
with a specific duration, as in Taylor (1980), but the contract duration is permitted to vary
across different groups of firms. For both specifications, the nominal and real rigidities
can be separately identified as long as the distribution of contract durations differs signif-
ically from the special case of Calvo contract with an exponential distribution. Finally,
we consider two distinct forms of indexation: “dynamic” indexation to lagged inflation, as
in Christiano, Eichenbaum, and Evans (2005); and “deterministic” indexation to the cen-
tral bank’s inflation objective, which is assumed to follow an exogenous path known to all
private agents.

To determine the structural characteristics of price-setting behavior in the context of
a stable monetary policy regime, we estimate this framework using two different macro
datasets: German data over the period 1975Q1 to 1998Q4 and U.S. data for the period
1983Q1 to 2003Q4. The German case provides an ideal setting for our analysis, because
the Bundesbank maintained a transparent and reasonably credible medium-term inflation
objective that declined gradually from 5 percent in 1975 to 2 percent in 1984 and remained
essentially constant thereafter. Thus, we can directly analyze the case of deterministic in-
dexation in terms of deviations of actual inflation from the Bundesbank’s explicit objective.
By comparison, U.S. inflation (measured using the non-farm business output price deflator)
remained quite stable around an average rate of about 3 percent during the mid-to-late
1980s and has remained stable at around 1.5 percent since the mid-1990s. In this case,
given the absence of an explicit U.S. inflation objective, we analyze the case of determinis-
tic indexation by allowing for a one-time break in the mean inflation rate in 1991Q1, based
on the findings of Levin and Piger (2004). For both the German and U.S. samples, we also
analyze the framework in terms of the level of inflation, corresponding to the case with no
deterministic indexation and hence effectively assuming a constant inflation objective.

Using simulation-based indirect inference methods to estimate the model, we find that
price-setting behavior in both Germany and the United States is well-characterized by
staggered contracts with an average duration of about two quarters, with indexation to
the central bank’s inflation objective but not to lagged inflation. Furthermore, the results
are reasonably similar regardless of whether we assume that contracts have random or fixed
duration. We also find that new price contracts exhibit very low sensitivity to marginal cost, corresponding to a relatively high degree of real rigidity involving both firm-specific inputs and strong curvature of the demand function. Finally, in each case, we confirm that the estimated model is not rejected by tests of overidentifying restrictions and that the implied autocorrelations provide a close match to those of an unrestricted vector autoregression. Evidently, backward-looking price-setting behavior (such as indexation to lagged inflation) is not needed to explain the aggregate data, at least in the context of a stable policy regime with a transparent and credible inflation objective.

Our empirical findings regarding the frequency of price adjustments are broadly consistent with recent evidence from firm-level surveys and micro price records. The microeconomic evidence also provides some indirect support for our focus on time-dependent rather than state-dependent specifications of price-setting behavior.

The remainder of this paper is organized as follows. Section 2 presents the generalized price-setting framework. Section 3 describes the data used in our analysis, while Section 4 reviews the estimation methodology. Section 5 reports the nominal rigidity parameter estimates for the case of deterministic indexation, while Section 6 interprets the estimated degree of real rigidity for this case. Section 7 considers the results obtained under the assumption of a constant inflation objective. Section 8 analyzes the hybrid NKPC using the same estimation methodology. Section 9 concludes.

2 The Generalized Price-Setting Framework

In this section we formulate a generalized price-setting framework that incorporates staggered nominal contracts of multiple durations while allowing contract duration to be either random or fixed. In the case of random duration contracts, our framework generalizes that

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8Caplin and Leahy (1997) and Dotsey, King, and Wolman (1999) have developed models of state-dependent price-setting, while Klenow and Kryvstov (2004) provide recent evidence on its limited role in generating aggregate inflation variability; see also recent work by Dotsey and King (2004).
of Calvo (1983) by allowing the probability of a price revision to depend on how long the existing contract has been in effect; that is, while assuming that every firm that resets its price faces the same \textit{ex ante} probability distribution of contract duration, we do not impose any restrictions on the shape of the hazard function.\textsuperscript{9} In the case of fixed duration contracts, our framework generalizes that of Taylor (1980) by allowing the contract duration to vary across different groups of firms; that is, each group of firms utilizes price contracts with a duration that is fixed and known at the start of the contract. Our framework also allows for “dynamic” indexation to lagged inflation as well as “deterministic” indexation to the central bank’s inflation objective.

Finally, our framework encompasses two sources of real rigidity. First, following Kimball (1995), each firm’s demand may exhibit a high degree of curvature (approximating a “kinked demand curve”) as a function of the firm’s price deviation from the average price level.\textsuperscript{10} Thus, when a firm is resetting its price contract, its optimal price will be relatively less sensitive to changes in the firm’s marginal cost. Second, the presence of fixed firm-specific inputs causes each firm’s marginal cost to vary with its level of output and hence dampens the sensitivity of new contract prices to an aggregate shock. For example, in considering a price hike in response to a particular shock, the firm recognizes that lower demand reduces its marginal cost, thereby partially offsetting the original rationale for raising its price.

Henceforth we will use the term “capital” to refer to the fixed factor in production, while the variable factor will be referred to as “labor.” Nevertheless, it should be emphasized that the fixed factor could include land as well as any overhead labor that cannot easily be adjusted in the short run. Furthermore, while our analysis abstracts from the influence of endogenous capital accumulation, the results of Eichenbaum and Fisher (2004) indicate that the degree of real rigidity is quantitatively similar for specifications with fixed capital and for specifications with an empirically reasonable magnitude of adjustment costs for investment.\textsuperscript{11}

\textsuperscript{9}Sheedly (2005) analyzes a random-duration contracting model in which the hazard function is restricted to be constant or monotonically decreasing—a restriction that is rejected for both samples considered here.\textsuperscript{10}See also Woglom (1982) and Ball and Romer (1990).\textsuperscript{11}Optimal price setting with firm-specific capital accumulation has recently been analyzed by Sveen and Wehne (2003), Christiano (2004), and Woodford (2004); see also Altig, Christiano, Eichenbaum, and Lindé (2004) and de Walque, Smets, and Wouters (2004).
2.1 The Market Structure

Consider a continuum of monopolistically competitive firms indexed by \( f \in [0, 1] \), each of which produces a differentiated good \( Y_t(f) \) using the following production function:

\[
Y_t(f) = A_t \bar{K}(f)^\alpha L_t(f)^{1-\alpha}.
\] (1)

Note that all firms have the same level of total factor productivity, \( A_t \). To ensure symmetry in the deterministic steady state, we also assume that every firm owns an identical capital stock, \( \bar{K}(f) = \bar{K} \).

A distinct set of perfectly competitive aggregators combine all of the differentiated products into a single final good, \( Y_t \), using the following technology:

\[
\int_0^1 G\left(\frac{Y_t(f)}{Y_t}\right) df = 1,
\] (2)

where the function \( G(\cdot) \) is increasing and strictly concave with \( G(1) = 1 \). Under this definition, the steady state of aggregate output, \( \bar{Y} \), is identical to the steady-state output of each individual firm, \( \bar{Y}(f) \).

Henceforth we use \( \eta \) to denote the steady-state elasticity of demand; that is, \( \eta = -G'(1)/G''(1) > 1 \). Furthermore, we use \( \epsilon \) to denote the relative slope of the demand elasticity around its steady-state value; that is, \( \epsilon = \eta G'''(1)/G''(1) + \eta + 1 \). Thus, the special case \( \epsilon = 0 \) corresponds to the Dixit-Stiglitz specification of constant demand elasticity, for which \( G(x) = x^{\eta/(\eta-1)} \).

Under these assumptions, each firm \( f \) faces the following implicit demand curve for its output as a function of its price \( P_t(f) \) relative to the price of the final good, \( P_t \):

\[
G'(Y_t(f)/Y_t) = \left(\frac{P_t(f)}{P_t}\right) \int_0^1 (Y_t(z)/Y_t)G'(Y_t(z)/Y_t) dz.
\] (3)

The concavity of \( G(\cdot) \) ensures that the demand curve is downward-sloping; that is, \( dY_t(f)/dP_t(f) < 0 \). The price index \( P_t \) can be obtained explicitly by multiplying both sides of equation (3) by the factor \( Y_t(f)/Y_t \) and then integrating over the unit interval:

\[
P_t = \int_0^1 (Y_t(z)/Y_t)P_t(z) dz.
\] (4)
Finally, the firm’s real marginal cost function $MC_t(f)$ is given as follows:

$$MC_t(f) = \frac{W_t}{(1-\alpha)P_tA_tK(f)^aL_t(f)^{-a}},$$  

(5)

where $W_t$ denotes the nominal wage rate.

### 2.2 The Duration of Price Contracts

In the case of random contract durations, every monopolistically competitive firm has the same *ex ante* hazard function that determines the probability that the firm is permitted to reset its price. Specifically, a firm $f$ signing a new contract in any given period $t$ faces the probability $\omega_j$ that its contract will last exactly $j$ periods, where $\omega_j \geq 0$ and $\sum_{j=1}^{J} \omega_j = 1$. Thus, the probability that the contract will continue for at least $j$ periods is given by $\Omega_j = \sum_{k=j}^{J} \omega_k$. Accordingly, among all price contracts in effect at a given point in time, the proportion of contracts that have been in effect for exactly $j$ periods is given by $\psi_j = \Omega_j / \sum_{k=1}^{J} \Omega_k$ for $j = 1, \ldots, J$. It should be noted that $\psi_j \geq 0$ for all $j$ and $\sum_{j=1}^{J} \psi_j = 1$.

This random-duration framework generalizes the particular case of Calvo-style contracts, in which each firm faces a constant probability $\xi$ of not revising its contract in any given period, and the maximum contract duration $J \to \infty$. Thus, in the Calvo formulation, the firm expects its contract to last exactly $j$ periods with probability $\omega_j = (1-\xi)\xi^{j-1}$, and the contract lasts at least $j$ periods with probability $\Omega_j = \xi^{j-1}$. Furthermore, the *ex post* distribution of outstanding contract durations is identical to the *ex ante* probability distribution; that is, $\psi_j = \omega_j$ for $j = 1, 2, \ldots$.

In the case of fixed contract durations, $\omega_j$ denotes the fraction of monopolistically competitive firms that sign price contracts with a duration of exactly $j$ periods ($j = 1, \ldots, J$), where again $\omega_j \geq 0$ and $\sum_{j=1}^{J} \omega_j = 1$. This framework generalizes the original formulation of Taylor (1980) in which the contract duration is identical for all firms, that is, the special case in which $\omega_J = 1$ for some particular value of $J$.

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12 Dixon and Kara (2005) have emphasized the distinction between *ex ante* probabilities and the realized distribution of contract durations, especially in comparing the mean duration implied by the Calvo vs. Taylor specifications.

13 In subsequent work, Taylor (1993) and Guerrieri (2002) allowed the fixed duration to vary across firms.
length \( j \) are evenly staggered, so that a fraction \( 1/j \) of the firms signing such contracts reset their contracts in any given period.\(^{14}\) Thus, among all price contracts in effect at a given point in time, the proportion of contracts with a duration of \( j \) periods is given by

\[
\psi_j = \frac{\Omega_j}{\sum_{k=1}^{J} \Omega_k} \quad (j = 1, \ldots, J),
\]

where \( \Omega_j = \omega_j/j \).

### 2.3 Indexation

Our framework allows for two distinct types of indexation. First, we allow for a general degree of “dynamic” indexation along the lines proposed by Christiano, Eichenbaum, and Evans (2005). This form of indexation may be particularly useful in accounting for a highly-autocorrelated inflation process that would be inconsistent with purely forward-looking price-setting behavior. On the other hand, this specification may be inconsistent with disaggregated evidence from surveys and micro price records, because dynamic indexation implies that prices are adjusted every period whereas the recent evidence indicates a lower frequency of price adjustment for a substantial fraction of goods and services.

In formal terms, when the firm’s price contract is not reoptimized, the firm faces an exogenous probability \( \delta \in [0, 1] \) that its price will be automatically adjusted to reflect the previous period’s aggregate gross inflation rate \( \Pi_t \); that is, \( P_t(f) = \Pi_t P_{t-1}(f) \). With probability \( 1 - \delta \), the firm’s price is simply adjusted by the steady-state gross inflation rate \( \bar{\Pi} \); that is, \( P_t(f) = \bar{\Pi} P_{t-1}(f) \).\(^{15}\) Thus, \( \delta = 1 \) represents the case in which all existing price contracts are dynamically indexed to lagged inflation, while \( \delta = 0 \) represents the case in which contracts are only indexed to steady-state inflation.

Second, contracts may exhibit “deterministic” indexation to the central bank’s current inflation objective, which is assumed to follow an exogenous path that is known to all private agents.\(^{16}\) Thus, allowing for both deterministic and dynamic indexation implies that with

\(^{14}\)As noted in Coenen and Levin (2004), these assumptions may be represented formally through an appropriate partition of the continuum of firms.

\(^{15}\)While highly stylized, the assumption of “static” indexation is innocuous for a steady-state inflation rate close to zero and helps maintain tractability in an empirical context with a non-zero mean inflation rate. For analysis of log-linear dynamics around a non-zero steady state without any automatic indexation, see Ascari (2003) for the case of random-duration contracts, Erceg and Levin (2003) for the case of fixed-duration contracts, and Cogley and Sbordone (2004) for empirical implications based on postwar U.S. data.

\(^{16}\)This specification may be viewed as a natural extension of Yun (1994) and Erceg, Henderson, and Levin
probability $\delta$, the firm’s price $P_t(f) = (\Pi_t^* / \Pi_{t-1}) \Pi_{t-1} P_{t-1}(f)$, where $\Pi_t^*$ denotes the central bank’s objective for the gross inflation rate. With probability $1 - \delta$, the firm’s new price is given by $P_t(f) = \Pi_t^* P_{t-1}(f)$.

While deterministic indexation is obviously a stylized assumption, this form of indexation preserves an important feature of the true non-linear price-setting framework, namely, that a perfectly anticipated and completely credible change in the central bank’s inflation objective need not involve any fluctuations in real marginal cost or any other real variables. In contrast, when the model is simply approximated around a constant inflation rate, any shift in the inflation objective must involve a corresponding change in real marginal cost, even if the shift in the inflation objective is completely transparent and credible. Thus, deterministic indexation avoids a potentially significant pitfall of the approach that has typically been followed in the literature.

2.4 The Optimal Price-Setting Decision

In period $t$, each firm resetting its contract chooses its new price $P_t(f)$ to maximize the firm’s expected discounted profits over the life of the contract,

$$E_t \left[ \sum_{i=0}^{J-1} \chi_i \kappa_{t,t+i} (P_{t+i}(f) Y_{t+i}(f) - W_{t+i} L_{t+i}(f)) \right] ,$$  

subject to the specific indexation process (which determines $P_{t+i}(f)$ as a function of the initial contract price $P_t(f)$) as well as the production function (1) and the implicit demand curve (3). The stochastic discount factor $\kappa_{t,t+i}$ can be obtained from the consumption Euler equation of the representative household.

If the price contract has random duration, then the coefficient $\chi_i$ indicates the probability that the price contract will still be in effect after $i$ periods; that is, $\chi_i = \Omega_{t+1}$ for $i = 0, \ldots, J-1$. If the price contract has a fixed duration, then the coefficient $\chi_i$ is simply an indicator function; that is, for a contract with a fixed duration of $j$ periods (for $j = 1, \ldots, J$), $\chi_i = 1$ for $i = 0, \ldots, j - 1$ and 0 otherwise.

(1999), who assumed indexation of all contracts to the constant steady-state inflation rate.
2.5 The Log-Linearization with Random Contract Duration

We now proceed to log-linearize the pricing equation and the aggregate price identity around the deterministic steady state. We use $\pi_t$ to denote the actual inflation rate, while $mc_t$ denotes the average real marginal cost across all firms in the economy (expressed as a logarithmic deviation from its steady-state value), and $y_t$ denotes the logarithmic deviation of aggregate output from steady state.

In the case of deterministic indexation, the inflation gap $\dot{\pi}_t$ is naturally defined as the deviation of actual inflation from the central bank’s objective; that is, $\dot{\pi}_t = \pi_t - \pi^*_t$. In contrast, in the absence of deterministic indexation, the inflation gap is simply the deviation of inflation from steady state: $\dot{\pi}_t = \pi_t - \bar{\pi}$. Moreover, because a fraction $\delta$ of existing price contracts are indexed to lagged inflation and to changes in the inflation objective, while the remaining portion are indexed solely to the current inflation objective, it is natural to define the quasi-difference of inflation as $\tilde{\pi}_t = \pi_t - \delta (\pi_{t-1} + \Delta \pi^*_t) - (1 - \delta) \pi^*_t$, or equivalently, as $\tilde{\pi}_t = \dot{\pi}_t - \delta \dot{\pi}_{t-1}$; that is, the quasi-difference of inflation can be expressed solely in terms of inflation gaps.

In the case of random contract durations, all firms signing new contracts at date $t$ set the same price. Thus, using $x_t$ to denote the logarithmic deviation of the new contract price from the aggregate price level, we obtain the following expression for the log-linearized optimal price-setting equation:

$$x_t = E_t \left[ \sum_{i=1}^{J-1} \Phi_i \tilde{\pi}_{t+i} + \gamma \sum_{i=0}^{J-1} \phi_i mc_{t+i} \right], \quad (7)$$

where the weights satisfy $\phi_i = \beta^i \Omega_{i+1} / \left( \sum_{k=0}^{J-1} \beta^k \Omega_{k+1} \right)$ and $\Phi_i = \sum_{k=i}^{J-1} \phi_k$, and $\beta$ denotes the household’s discount factor.

The coefficient $\gamma$ in equation (7) determines the sensitivity of new price contracts to aggregate real marginal cost. In particular, as shown by Eichenbaum and Fisher (2004), this coefficient can be expressed as the product of two components; that is, $\gamma = \gamma_d \cdot \gamma_{mc}$, where

$$\gamma_d = \frac{\eta - 1}{\epsilon + \eta - 1}, \quad (8)$$

$$\gamma_{mc} = \frac{1}{1 + \frac{\alpha}{\eta} \eta \gamma_d}. \quad (9)$$
It should be noted that the coefficient $\gamma_d$ depends solely on the relative curvature of the firm’s demand function, and has a value of unity in the special case with constant demand elasticity; that is, $\gamma_d = 1$ when $\epsilon = 0$. The coefficient $\gamma_{mc}$ reflects the degree to which the firm’s relative price influences its marginal cost, and has a value of unity in the special case with no fixed factors; that is, $\gamma_{mc} = 1$ when $\alpha = 0$.

The log-linearized aggregate price identity can be expressed as follows:

$$\sum_{i=0}^{J-1} \psi_{i+1} x_{t-i} = \sum_{i=0}^{J-2} \Psi_{i+1} \tilde{\pi}_{t-i},$$

(10)

where $\Psi_i = \sum_{k=i+1}^{J} \psi_k$.

### 2.6 The Log-Linearization with Fixed Contract Duration

For the case of fixed-duration contracts, let $x_{j,t}$ indicate the logarithmic deviation of the new contract price of duration $j$ from the aggregate price level. Then the log-linearized price-setting equation for this type of contract can be expressed as follows:

$$x_{j,t} = E_t \left[ \sum_{i=1}^{j-1} \Lambda_{j,i} \tilde{\pi}_{t+i} + \gamma \sum_{i=0}^{j-1} \lambda_{j,i} mc_{t+i} \right],$$

(11)

where $\lambda_{j,i} = \beta^i / \sum_{k=0}^{j-1} \beta^k$ and $\Lambda_{j,i} = \sum_{k=0}^{j-1} \lambda_{j,k}$.

The aggregate price level depends on all of the price contracts in effect at date $t$; thus, recalling that $\Omega_j = \omega_{j}/j$ in the case of fixed-duration contracts, we obtain the following expression for the aggregate price identity:

$$\sum_{i=0}^{J-1} \sum_{j=i+1}^{J} \Omega_j x_{j,t-i} = \sum_{i=0}^{J-2} \sum_{j=i+2}^{J} (j-i-1) \Omega_j \tilde{\pi}_{t-i}.$$

(12)

### 2.7 Implied Inflation Dynamics and Identification

In discussing the implications of our generalized price-setting framework with staggered contracts of multiple durations for inflation dynamics, we focus on the case of random-duration contracts. In this context, recall that $\chi_i = \Omega_i + 1$ is the probability (at the start of the contract) that the contract will have a duration of at least $i$ periods. In the case of Calvo contracts, $\chi_i = \xi^i$ and $J \to \infty$; thus, the probability that the contract lasts exactly
periods is given by \( \omega_j = (1 - \xi) \xi^{j-1} \). Substituting these relations yields the formulas

\[ \phi_i = (1 - \beta \xi) \beta \xi^i, \quad \Phi_i = \beta \xi^i, \quad \psi_i = (1 - \xi) \xi^{i-1}, \text{ and } \Psi_i = \xi^i. \]

Hence, the analytical simplicity of the Calvo specification occurs precisely because of the proportionality of the coefficients of the log-linearized price equations; that is, Calvo contracts imply that \( \phi_i \) is proportional to \( \Phi_i \) and that \( \psi_i \) is proportional to \( \Psi_i \). More specifically, by using lag operator notation and cancelling terms in equation (10), we find that \( x_t \) (the deviation of new price contracts from the aggregate price level) is proportional to the current quasi-difference of the inflation rate \( \pi_t \):

\[ (1 - \xi) x_t = \xi \pi_t \tag{13} \]

Furthermore, by substituting this relation into equation (7) and recalling that \( \pi_t = \pi_t - \delta \pi_{t-1} \), we find that in this case the generalized price-setting equation simplifies to the hybrid New Keynesian Phillips curve:

\[ \pi_t = \frac{\delta}{1 + \delta} \pi_{t-1} + \frac{\beta}{1 + \delta} E_t[\pi_{t+1}] + \gamma \frac{(1 - \xi)(1 - \beta \xi)}{(1 + \delta) \xi} m_{ct}. \tag{14} \]

Evidently, in the special case of Calvo contracts, it is possible to obtain a simple expression for the price-setting relation that only involves the aggregate inflation rate and real marginal cost, without any explicit reference to current or lagged price contracts. And in this case, as noted previously, the real rigidity parameter \( \gamma \) is not separately identified from the nominal rigidity parameter \( \xi \).

In contrast, when the duration of price contracts does not exhibit a purely exponential rate of decay, then the price-setting equation (7) and the aggregate price identity (10) are both required for a complete representation of the system of price determination. Thus, the generalized staggered contracts model implies that current inflation is related to its own lagged values (even apart from the influence of dynamic indexation) as well as to expected inflation at longer horizons (not just one period ahead):

\[ \pi_t = -\Psi^{-1} \sum_{i=1}^{J-2} \Psi_{i+1} \pi_{t-i} + \Psi^{-1} \sum_{i=0}^{J-1} \sum_{j=1}^{J-1} \psi_{i+1} \Phi_j E_t[\pi_{t+j-i}] + \gamma \Psi^{-1} \sum_{i=0}^{J-1} \sum_{j=0}^{J-1} \psi_{i+1} \phi_j E_t[mc_{t+j-i}], \tag{15} \]
Furthermore, the nominal and real rigidity parameters can be separately identified because these parameters have distinct implications for aggregate inflation dynamics. In particular, the ability to separately identify the nominal and real rigidities depends crucially on the fact that the generalized staggered contracts model exhibits more complex dynamics than the pure Calvo specification. And in practice, this means that the precision of these estimates will depend on the extent to which we can reject the special case in which the distribution of contract durations exhibits exponential decay.

3 The Data

In this section, we describe the two datasets used in our empirical analysis of the generalized price-setting framework.

3.1 Germany, 1975-1998

German macroeconomic data for the period 1975-1998 provides a virtually ideal setting for determining the structural characteristics of price-setting, because the Bundesbank maintained a reasonably transparent and credible medium-term inflation objective over this period. In particular, in explaining the derivation of each annual target for money growth, the Bundesbank communicated its assumptions regarding the level of inflation over the medium run, set in the broader context of the ultimate goal of price stability.¹⁷

The upper-left panel of Figure 1 depicts the evolution of actual inflation and the Bundesbank’s medium-term inflation objective over the period 1975 to 1998. In the previous year, GDP price inflation had reached a transitory peak of about 8 percent in the wake of the collapse of the Bretton Woods regime and the first OPEC oil price shock. By 1975, however, inflation stabilized around the Bundesbank’s medium-term inflation objective of about 5 percent. Inflation subsequently declined fairly gradually through the late 1970s and early 1980s, roughly in parallel with reductions in the Bundesbank’s medium-term objective. From about 1985 through the advent of the European Monetary Union, the

¹⁷Starting in 1976, the Bundesbank’s medium-term inflation objective was published in each Annual Report as well as in various issues of the Monthly Bulletin. For further details, see Schmid (1999) and Gerberding, Seitz, and Worns (2004).
Note: Inflation is measured as the annualized quarter-on-quarter change in the logarithm of the GDP price deflator. The inflation gap is defined as the deviation of inflation from the Bundesbank’s medium-term inflation objective. The labor share is constructed as the ratio of total compensation (including imputed labor income of self-employed workers) to nominal GDP. The markup gap is defined as the deviation of the logarithm of the labor share from a linear trend.

inflation objective remained essentially constant at 2 percent; actual inflation exhibited an average level fairly close to this objective, with only one large deviation in the early 1990s during the process of German unification. Our empirical investigation proceeds by fitting the deviations of actual inflation from the Bundesbank’s medium-term inflation objective; this “inflation gap” is shown in the upper-right panel of Figure 1.

The labor share serves as our benchmark proxy for real marginal cost. In measuring the labor share, it is important to account for the significant role of self-employed workers in the
German economy. In the absence of direct measures of labor compensation for self-employed workers, we follow the fairly standard approach of computing the labor share by taking the compensation of employees (which does not include self-employed workers), multiplying this figure by the ratio of total employment (including self-employed workers) to the number of employees, and then dividing by nominal GDP. In effect, this procedure uses the average compensation rate of employees to impute the labor compensation of self-employed workers.

The lower-left panel of Figure 1 depicts the evolution of the German labor share, which
exhibits a clear downward trend over the sample period. This pattern is similar to that observed in other continental European economies such as France and Italy, apparently reflecting a gradual decline in union bargaining power as well as other structural factors. Since our analytical framework follows the standard New Keynesian view that prices adjust in response to deviations of the actual markup from a desired level, we interpret the low-frequency movement of the labor share as a deterministic trend in the desired markup. Thus, our price-setting framework is estimated using the detrended labor share—henceforth referred to as the markup gap—as depicted in the lower-right panel of Figure 1.

In performing sensitivity analysis, we consider several alternative proxies for real marginal cost, each of which is depicted in Figure 2. The upper-right panel shows two measures of the output gap, which have been constructed from real GDP (shown in the upper-left panel) using linear detrending and Hodrick-Prescott filtering, respectively. The lower-left panel depicts the ratio of employee compensation to nominal GDP. This measure—henceforth referred to as the uncorrected labor share—implicitly attributes all of the income of self-employed workers as compensation to capital rather than labor. The behavior of the detrended series (shown in the lower-right panel) is broadly similar to that of the benchmark series, but the deviation from trend is much larger in the mid-1970s; given that this deviation is not accompanied by substantial movement in inflation, we shall see below that the uncorrected labor share implies an even higher degree of real rigidity than the benchmark series.

3.2 United States, 1983-2003

As shown in the upper-left panel of Figure 3, the U.S. inflation rate (as measured by the price deflator for non-farm business output) exhibited a distinct downward shift in the early 1990s. In particular, while inflation was quite stable around an average rate of 3 percent during the mid-to-late 1980s, inflation has subsequently averaged about 1.5 percent. Because the Federal Reserve did not pursue an explicit longer-term inflation objective over this period, we proceed by assuming that the inflation objective was subject to a one-time

---

Figure 3: U.S. Inflation and Markup Gaps, 1983-2003

Note: Inflation is measured as the annualized quarter-on-quarter change in the logarithm of the GDP price deflator. The labor share is constructed as the ratio of total compensation in the non-farm business sector to nominal GDP.

discrete break in 1991Q1—the date chosen in the analysis of Levin and Piger (2004)—and then demean each of the two subsamples. The resulting “inflation gap” is depicted in the upper-right panel of the figure.

In contrast to the German data, the U.S. labor share does not exhibit any marked time trend over the relevant sample period, as shown in the lower-left panel. In this case, therefore, we proceed by assuming a constant value of the desired markup, yielding the benchmark “markup gap” shown in the lower-right panel.
4 Estimation Methodology

Our empirical analysis essentially follows the approach of Coenen and Wieland (2005). In the first stage, we estimate an unconstrained VAR model that provides an empirical description of the dynamics of the inflation gap, the markup gap, and the output gap. In the second stage, we employ simulation-based indirect inference methods to estimate the structural price-setting equations, using the unconstrained VAR as the auxiliary model. In effect, this method determines the parameters of the structural model by matching its reduced form—which constitutes a constrained VAR—as closely as possible with the unconstrained VAR.\cite{19}

In the remainder of this section, we compare our procedure with alternative approaches that have been employed in the literature, and then describe the estimation methodology in further detail.

4.1 Comparison with Alternative Approaches

Unlike most of the literature on estimating NKPCs, standard method-of-moments procedures cannot be applied to our generalized price-setting framework due to the presence of unobserved variables (namely, the new contracts signed each period). Furthermore, since each contract price depends on expected future markup gaps, we need to specify how these gaps are determined. To avoid imposing any additional restrictions, we simply take the markup gap and output gap equations from the unconstrained VAR and combine these with the structural price-setting equations; we refer to the combined set of equations as the “structural model” even though only part of the model is truly structural.\cite{20}

Our estimation methodology has some appealing features compared with several other commonly-employed procedures. For example, one alternative approach is to specify a complete structural model and estimate its parameters by matching some of the implied impulse response functions (IRFs) to those of an identified VAR model.\cite{21} In contrast, our

\cite{19}The method of indirect inference was proposed by Smith (1993) and Gouriéroux, Monfort and Renault (1993); see also Gouriéroux and Monfort (1996).
\cite{20}This limited-information approach follows Taylor (1993) and Fuhrer and Moore (1995), and is similar in spirit to the approach of Sbordone (2002).
\cite{21}Recent examples of this approach include Rotemberg and Woodford (1997), Christiano et al. (2005),
procedure matches the implications of the structural model to those of an unconstrained VAR, thereby avoiding the need to impose potentially controversial identifying assumptions on the auxiliary model. Furthermore, our procedure essentially matches all of the sample autocorrelations and cross-correlations rather than a limited set of characteristics of the data.

Another alternative approach involves the use of full-information methods to estimate a complete structural model. Nevertheless, one potential pitfall of that approach is that the price-setting parameter estimates could be sensitive to misspecifications in other aspects of the model—a particularly important issue in this case due to the lack of consensus about which labor market rigidities are relevant in determining the behavior of the markup gap.

4.2 Details of the Estimation Procedure

We begin by using ordinary least-squares to estimate an unconstrained VAR involving the inflation gap, the markup gap, and the output gap. We then proceed to use this model as a benchmark for conducting indirect inference on the structural model, which consists of the generalized price-setting framework combined with the markup gap and output gap equations taken from the unconstrained VAR. In our empirical analysis, the optimal price-setting equation includes an exogenous white-noise disturbance that may reflect shifts in sales tax rates or stochastic variation in the desired markup.

For a sample of length $T$, the vector of parameter estimates of the unconstrained VAR is denoted by $\hat{\zeta}_T$, while the estimated covariance matrix of these parameters is denoted by $\hat{\Sigma}_T(\hat{\zeta}_T)$. It should be noted that the vector $\hat{\zeta}_T$ includes not only the VAR coefficients but also the variances and contemporaneous correlations of the innovations. The unconstrained VAR is specified with three lags of each variable; this specification yields serially uncorrelated residuals (based on the Ljung-Box Q statistic) and corresponds to the reduced-form VAR.

For recent examples of full-information estimation, see Schorfheide (2000), Smets and Wouters (2003), and Onatski and Williams (2004).

Of course, when the output gap is used as the proxy for real marginal cost, the unconstrained model is simply a bivariate VAR involving the inflation gap and the output gap, and the structural model consists of the generalized price-setting framework and the output gap equation from the unconstrained VAR.

See Clarida et al. (1999) and Woodford (2003).
representation of the structural model when price contracts have a maximum duration of four quarters.\footnote{For both the German and U.S. samples, AIC and BIC each prescribe a VAR lag order of only 1 or 2 lags. As with spectral density estimation and unit root tests, we believe it is prudent to use a somewhat higher lag order of 3, because the bias associated with using an insufficient lag order tends to be much more harmful than the loss of precision associated with using an excessive lag order; cf. Ng and Perron (2001). In fact, sensitivity analysis (available upon request) indicates that a VAR(2) yields somewhat lower estimates of the mean contract duration and the real rigidity parameter whereas a VAR(4)–the specification used in various other studies–moves these estimates in the opposite direction.}

The vector of structural parameters, $\theta$, includes the distribution of contract durations ($\omega_j$ for $j = 1, ..., 4$), the sensitivity of new contracts to aggregate real marginal cost ($\gamma$), and the standard deviation of the white-noise disturbance to the optimal price-setting equation ($\sigma_\epsilon$). The distribution of contract durations is estimated subject to the constraint that these parameters are non-negative and sum to unity. Finally, rather than estimating the discount factor, we simply calibrate $\beta = 0.9925$, corresponding to an annualized steady-state real interest rate of about 3 percent.

For any particular vector of structural parameters $\theta$, we confirm that the model has a unique linear rational expectations solution and then obtain its reduced-form VAR representation using the AIM algorithm of Anderson and Moore (1985). Using this reduced-form model, we generate “artificial” time series of length $S$ for the endogenous variables, namely, the relative contract prices, the inflation gap, the markup gap, and the output gap.\footnote{To simulate the model, we employ a Gaussian random-number generator for the disturbances, and we use steady-state values as initial conditions for the endogenous variables; the first few years of simulated data are excluded from the sample used for indirect inference to ensure that the results are not influenced by these particular initial conditions. The effective sample size is $S = 100T$.} We then fit the latter three randomly-generated series with an unconstrained VAR model that is isomorphic to the one applied to the observed data. The vector of fitted VAR parameters is denoted by $\hat{\zeta}_S(\theta)$ because these VAR parameters depend on the particular values of the structural parameters $\theta$ as well as the restrictions of the structural model and the sample size $S$ of the simulated data.

We then use a numerical optimization algorithm to determine the set of structural parameters that maximizes the fit between the simulation-based VAR parameters and those of the unconstrained VAR of the observed data. In particular, the estimated value of $\theta$ minimizes the following criterion function:
\[ Q_{S,T}(\theta) = \left( \hat{\zeta}_T - \hat{\zeta}_S(\theta) \right)' S' \left[ S \hat{\Sigma}_T(\hat{\zeta}_T) S' \right]^{-1} S \left( \hat{\zeta}_T - \hat{\zeta}_S(\theta) \right), \tag{16} \]

where \( S \) is the matrix of zeros and ones that selects the elements of \( \hat{\zeta}_T \) that correspond to the inflation equation of the unconstrained VAR.\(^{27}\)

Because this criterion function employs the optimal weighting matrix, the resulting estimator of \( \theta \) is asymptotically efficient. In particular, under certain regularity conditions (including the assumption that the sample size ratio \( S/T \) converges to a constant \( q \) as \( T \to \infty \)), this estimator is consistent and has the following asymptotic normal distribution:

\[ \sqrt{T}(\hat{\theta}_{S,T} - \theta_0) \xrightarrow{d} N(0, (1 + q^{-1})(Z' S' \left[ S \Sigma(\zeta_0) S' \right]^{-1} S Z)^{-1}), \tag{17} \]

where \( \theta_0 \) is the probability limit of \( \hat{\theta}_{S,T} \); \( \zeta_0 \) is the plim of \( \hat{\zeta}_T \); \( \Sigma(\zeta_0) \) is the plim of \( \hat{\Sigma}_T(\hat{\zeta}_T) \); \( z(\theta_0) \) is the plim of \( \hat{\zeta}_S(\theta_0) \) as \( S \to \infty \); and \( Z = (\partial z(\theta_0)/\partial \theta') \).\(^{28}\)

5 Gauging the Degree of Nominal Rigidity

We now proceed to gauge the degree of nominal rigidity obtained under the assumption of deterministic indexation to the central bank’s inflation objective. In addition to examining the estimated degree of dynamic indexation and the estimated distribution of price contract durations, we consider evidence on the model’s goodness-of-fit, which confirms that this framework provides a reasonably close match to the data.

5.1 Benchmark Estimates

The first column of Table 1 reports the estimated degree of dynamic indexation of German and U.S. price contracts when the model is estimated using the benchmark series for the inflation gap and markup gap. For both the random-duration and fixed-duration specifications, the point estimate for the dynamic indexation parameter \( \delta \) is at zero (the lower

\(^{27}\)This choice of the selection matrix \( S \) is useful for alleviating the computational burden of our estimation procedure. In principle, all elements of \( \hat{\zeta}_T \) could be included in the estimation. However, our estimation results are unlikely to change because the markup gap and output gap equations in our structural model are taken from the unconstrained VAR itself. The finding that the autocorrelation functions of the markup gap and the output gap implied by the estimated structural model are virtually identical to those implied by the unconstrained VAR is reassuring in this respect.

\(^{28}\)Further discussion of these asymptotic properties may be found in the papers cited in footnote 19; a useful summary is also provided in the appendix to the working paper version of Coenen and Wieland (2005).
## Table 1: Benchmark Estimates of Nominal Rigidity

<table>
<thead>
<tr>
<th></th>
<th>Dynamic Indexation</th>
<th>Distribution of Contract Durations</th>
<th>Mean Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\omega_1$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td><strong>Germany, 1975-1998</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random-Duration Contracts</td>
<td>0.00</td>
<td>0.55</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>[0.25]</td>
<td>(0.12)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Fixed-Duration Contracts</td>
<td>0.00</td>
<td>0.33</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>[0.25]</td>
<td>(0.08)</td>
<td>(0.07)</td>
</tr>
<tr>
<td><strong>United States, 1983-2003</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random-Duration Contracts</td>
<td>0.00</td>
<td>0.43</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>[0.31]</td>
<td>(0.07)</td>
<td>—</td>
</tr>
<tr>
<td>Fixed-Duration Contracts</td>
<td>0.00</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>[0.31]</td>
<td>(0.06)</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: This table reports the estimated degree of dynamic indexation ($\delta$) and the distribution of contract durations ($\omega_j$) for each specification of the generalized price-setting framework (that is, random or fixed durations), obtained using the benchmark inflation gap and the benchmark markup gap for each sample. The upper bound of the 95th percentile is enclosed in square brackets, while estimated standard errors are given in parentheses.

The absence of dynamic indexation is consistent with the conclusions of Christiano, Eichenbaum, and Evans (2005), who found that dynamic price indexation is not essential for matching the impulse response function of an identified U.S. monetary policy shock. The U.S. results are also remarkably similar to those obtained using Bayesian analysis of dynamic stochastic general equilibrium (DSGE) models.\(^{30}\)

The remaining columns of Table 1 indicate the estimated distributions of German and U.S. price contract durations obtained using the benchmark inflation gap and the benchmark markup gap series. For both the random-duration and fixed-duration specifications, the

---

\(^{29}\)Because the point estimate is on the edge of the admissible region, we determine the upper bound of the confidence interval by finding the value of the dynamic indexation parameter for which the difference in the minimized criterion function exceeds the 95 percent confidence level.

\(^{30}\)See Levin, Onatski, Williams, and Williams (2005). Smets and Wouters (2003) obtained higher estimates of the dynamic indexation parameter in analyzing synthetic euro area data, but that finding may reflect aggregation effects as well as time variation in the inflation objectives of the individual countries that ultimately joined EMU.
estimated distribution of contract durations corresponds to a relatively moderate degree of nominal rigidity—with an average duration of about 2-3 quarters—that is broadly consistent with evidence from firm-level surveys and micro price records regarding the frequency of price adjustment. Furthermore, while the mean duration of U.S contracts is estimated to be a bit longer than for German contracts, the differences are not statistically significant.\footnote{For the U.S. price contract durations, the estimation procedure hits the non-negativity constraint on $\omega_2$; thus, the results are reported under the restriction that $\omega_2 = 0$.}

For the random-duration specification, the distribution of contract durations is markedly different from the exponential pattern implied by Calvo-style contracts. In the German case, for example, Calvo contracts with a mean duration of slightly less than two quarters would imply probabilities of roughly 0.52, 0.27, 0.14, and 0.07 (for durations of one to four quarters, respectively), whereas a duration of four quarters has a substantially higher probability in the estimated distribution.\footnote{For the German sample, the Calvo restriction can be rejected at the 90 percent confidence level. Instead, the German data are consistent with a “truncated Calvo” specification in which every contract is reoptimized when it reaches the maximum duration of four quarters; that is, for the estimated mean duration of 1.95 quarters, the truncated Calvo model would imply probabilities of 0.48, 0.23, 0.11, and 0.18 for durations of one to four quarters, respectively.} The contrast is even more dramatic for the U.S. sample: in this case, the estimated distribution is strikingly bimodal, with probabilities close to 50 percent on durations of one and four quarters.

It is interesting to note that the distribution of contract durations is noticeably longer for the fixed-duration specification. In this case, each individual firm is assumed to know exactly how long its price contract will remain in effect, whereas the random-duration specification assumes that all new price contracts signed each period have the same \textit{ex ante} expected duration. Thus, to match the observed sensitivity of aggregate inflation to the one-year-ahead markup gap, the fixed-duration specification must incorporate a somewhat larger share of four-quarter contracts and a correspondingly smaller share of one-quarter contracts compared with the random-duration specification.

Finally, as shown in Table 2, the estimated distribution of contract durations is not sensitive to alternative proxies for the German markup gap. As discussed in Section 3.1, these proxies include an alternative markup gap (that is, a measure of the labor share that omits the imputed labor income of self-employees) as well as linearly-detrended and
Table 2: Robustness of German Results to Alternative Proxies for Real Marginal Cost

<table>
<thead>
<tr>
<th>Distribution of Contract Durations</th>
<th>Mean Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td>A. Random-Duration Contracts</td>
<td></td>
</tr>
<tr>
<td>Alternative</td>
<td>0.54</td>
</tr>
<tr>
<td>Markup Gap</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.49</td>
</tr>
<tr>
<td>(Linear Trend)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.44</td>
</tr>
<tr>
<td>(HP Trend)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>B. Fixed-Duration Contracts</td>
<td></td>
</tr>
<tr>
<td>Alternative</td>
<td>0.34</td>
</tr>
<tr>
<td>Markup Gap</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.33</td>
</tr>
<tr>
<td>(Linear Trend)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.33</td>
</tr>
<tr>
<td>(HP Trend)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

Note: This table reports the distribution of contract durations for each specification of the generalized price-setting framework (that is, random or fixed durations) using the benchmark inflation gap along with several alternative proxies for real marginal cost. Estimated standard errors are given in parentheses.

HP-filtered measures of the output gap. In all cases, the estimated mean contract duration remains at about two quarters, and the individual results are quite close to the corresponding benchmark estimates reported in Table 1.

5.2 Consistency with the Data

As discussed earlier, our estimation procedure is aimed at matching the reduced-form implications of the structural model to those of an unconstrained VAR. Thus, a natural starting point for evaluating the goodness-of-fit of the structural model is to compare its implied autocorrelations with the sample autocorrelations of the observed time series.\(^{33}\) We also test the null hypothesis of no serial correlation in the implied disturbances to the optimal

price-setting equation, which is our maintained assumption in estimating the model.

According to both metrics, the generalized price-setting framework performs quite well in fitting the characteristics of the macroeconomic data. For example, Figure 4 depicts correlograms for the random-duration specification estimated using the benchmark inflation gap and markup gap series. For the German sample, the autocorrelations of inflation implied by the structural model are virtually indistinguishable from those of the observed data, while the contract price shocks generally exhibit negligible autocorrelation—a finding
Table 3: Testing the Overidentifying Restrictions

<table>
<thead>
<tr>
<th></th>
<th>Random-Duration Contracts</th>
<th>Fixed-Duration Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Germany, 1975-1998</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark Markup Gap</td>
<td>0.39</td>
<td>0.41</td>
</tr>
<tr>
<td>Alternative Markup Gap</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Output Gap (Linear Trend)</td>
<td>0.14</td>
<td>0.25</td>
</tr>
<tr>
<td>Output Gap (HP Trend)</td>
<td>0.06</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>United States, 1983-2003</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark Markup Gap</td>
<td>0.08</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: This table indicates the probability that the overidentifying restrictions are consistent with each specification of the generalized price-setting framework, estimated using the benchmark inflation gap and the specified proxy for real marginal cost.

which is confirmed by portmanteau tests for serial correlation. For the U.S. sample, the correlograms for the U.S. sample exhibit somewhat greater variability but generally lie well within the asymptotic confidence bands.\(^\text{34}\)

A more formal means of evaluating the structural model is to test whether the overidentifying restrictions of the model are consistent with the data. The degrees of freedom of the overidentification test depends on the number of free parameters in the structural model compared with the unconstrained VAR. When the structural model is estimated using one of the markup gap series as a proxy for real marginal cost, the model is matched to an trivariate VAR involving the markup gap, inflation gap, and output gap; in this case, the test of overidentifying restrictions has seven degrees of freedom. When the structural model is estimated using the output gap as the proxy for real marginal cost, then the corresponding unconstrained model is a bivariate VAR involving the inflation gap and the output gap, and the overidentification test has three degrees of freedom.

The p-values obtained from these overidentification tests are reported in Table 3. For both the German and U.S. samples, the overidentifying restrictions are not rejected for either the random-duration or fixed-duration specification when the model is estimated

\(^{34}\text{See Coenen (2005) for a detailed discussion of the methodology used in computing the asymptotic confidence bands for the estimated autocorrelation functions.}\)
using the benchmark inflation gap and markup gap series. Furthermore, it is apparent that these results are not simply due to lack of statistical power, because the overidentifying restrictions are indeed rejected at the 95 percent confidence level when the uncorrected German markup gap is used as the proxy for real marginal cost. The latter result arises from the noticeably weaker link between inflation and the uncorrected markup gap, which is markedly lower than our benchmark measure in the early and late portions of our sample.

6 Interpreting the Degree of Real Rigidity

While our generalized price-setting framework directly identifies the distribution of nominal contract durations, the degree of real rigidity is summarized by a single composite parameter, $\gamma$. We now consider the implications of the estimated value of $\gamma$—corresponding to a relatively high degree of real rigidity—in terms of the underlying structural parameters of the firm’s production and demand functions. In evaluating the degree of real rigidity, the model with no firm-specific inputs and a constant elasticity of demand provides a natural benchmark, because in this case $\gamma = \gamma_d = \gamma_{mc} = 1$; that is, a one percent increase in real marginal cost causes a one percent rise in the level of new price contracts.

Table 4 indicates that new price contracts exhibit relatively low sensitivity to real marginal cost. For example, $\gamma$ is only about 0.027 for the random-duration specification estimated for the German sample using the benchmark inflation gap and markup gap series; the estimated value is even smaller for the U.S. sample. Furthermore, equation (8) suggests that both firm-specific inputs and strong curvature of the demand function are needed to generate the estimated degree of real rigidity.

As indicated in Section 2, the sensitivity of new price contracts to aggregate marginal cost ($\gamma$) depends on the share parameter ($\alpha$), the steady-state demand elasticity ($\eta$), and the relative slope of the demand elasticity at steady state ($\epsilon$). Thus, we now investigate how the implied degree of real rigidity varies with each of these underlying structural parameters.

To explore the role of firm-specific fixed factors, we consider two distinct values for the share parameter $\alpha$. With the fairly standard calibration of $\alpha = 0.3$, the firm-specific fixed factor (capital) accounts for 30 percent of total cost while the variable input (labor)
Table 4: The Estimated Degree of Real Rigidity

<table>
<thead>
<tr>
<th></th>
<th>Random-Duration Contracts</th>
<th>Fixed-Duration Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Germany, 1975-1998</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.027</td>
<td>0.014</td>
</tr>
<tr>
<td>Markup Gap</td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Alternative</td>
<td>0.008</td>
<td>0.004</td>
</tr>
<tr>
<td>Markup Gap</td>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Output Gap (Linear Trend)</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Output Gap (HP Trend)</td>
<td>0.028</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
<tr>
<td><strong>United States, 1983-2003</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>Markup Gap</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Note: For each specification of the generalized price-setting framework, this table reports the estimated real rigidity parameter (γ) obtained using the benchmark inflation gap and the specified proxy for real marginal cost. Estimated standard errors are given in parentheses.

accounts for 70 percent of total cost. The alternative calibration α = 0.6 may be interpreted as reflecting a much higher degree of capital intensity in production, or (perhaps more realistically) the extent to which a substantial fraction of the labor input should also be viewed as a firm-specific fixed factor.

Reflecting the degree of empirical controversy regarding the steady-state demand elasticity, we consider values of η ranging from 5 to 20. Since the steady-state markup rate is equal to η/(η − 1), the bottom of this range corresponds to a steady-state markup rate of 25 percent, while the top of the range implies a 5 percent markup rate. With an even more severe paucity of evidence about the value of ϵ, we follow Eichenbaum and Fisher (2004) in considering three distinct specifications for this parameter: ϵ = 0, corresponding to the Dixit-Stiglitz specification of constant demand elasticity; ϵ = 10, based on the findings of Bergin and Feenstra (2000); and ϵ = 33, based on the analysis of Kimball (1995) and Chari, Kehoe, and McGrattan (2000).

Each panel of Figure 5 depicts the implied value of γ for alternative values of η and ϵ for
Figure 5: Accounting for the Estimated Degree of Real Rigidity

A. 30 Percent Cost Share of Firm-Specific Inputs

B. 60 Percent Cost Share of Firm-Specific Inputs

Note: Each panel indicates the implied degree of real rigidity ($\gamma$) corresponding to alternative combinations of the steady-state demand elasticity ($\eta$) and the curvature of demand ($\epsilon$); the upper panel depicts these results for $\alpha = 0.3$, while the lower panel gives corresponding results for $\alpha = 0.6$. The horizontal line at $\hat{\gamma} = 0.027$ indicates the parameter estimate for the random-duration model obtained using the German benchmark inflation gap and markup gap series, while the dotted lines denote the 95% confidence bands associated with this estimate.
a particular value of the share parameter $\alpha$. For ease of reference, the figure also indicates the estimated value of $\hat{\gamma} = 0.027$ and the corresponding 95 percent confidence interval that we obtained for the random-duration contract model using the German benchmark inflation gap and markup gap series.

When firm-specific fixed inputs account for 30 percent of total cost ($\alpha = 0.3$), no plausible combination of values of $\eta$ and $\epsilon$ can account for the estimated value of $\gamma$. For example, with a constant demand elasticity and a steady-state markup rate of 10 percent (that is, $\epsilon = 0$ and $\eta = 11$), the implied value of $\gamma$ is about 0.14. Even with very strong curvature of the demand function ($\epsilon = 33$), the implied value of $\gamma$ is several times larger than the benchmark estimate $\hat{\gamma}$.

In contrast, when firm-specific factors account for 60 percent of total cost ($\alpha = 0.6$), the model-implied value of $\gamma$ lies within the 95 percent confidence interval whenever the steady-state demand elasticity is sufficiently high. For example, with a constant demand elasticity ($\epsilon = 0$), the value of $\gamma = 0.03$ is obtained for $\eta = 16$, corresponding to a steady-state markup rate of about 6 percent. Furthermore, the specific value of $\epsilon$ is not very important in this case, because the value of $\gamma$ is insensitive to $\epsilon$ when $\eta$ and $\alpha$ are relatively large.

Although our empirical results are reasonably robust to the choice of proxy for real marginal cost (e.g., the labor share or the output gap), it is important to recognize that each of these variables is likely to involve fairly large and persistent measurement errors. Thus, before drawing definitive conclusions about the likely combination of underlying structural parameters, it is important to gauge the extent to which the estimated real rigidity parameter may exhibit downward bias due to the mismeasurement of real marginal cost. These results also underscore the need for further work in finding better proxies for real marginal cost, or alternatively, identifying instrumental variables that are orthogonal to the measurement errors that are likely to be present in the observed series.

7 The Role of the Time-Varying Inflation Objective

Our discussion thus far has focused on the estimation results obtained using the benchmark inflation gap (that is, $\hat{\pi}_t = \pi_t - \pi^*_t$) which corresponds to the assumption of determinis-
Table 5: Estimates of Nominal Rigidity Parameters in the Absence of a Time-Varying Inflation Objective

<table>
<thead>
<tr>
<th>Dynamic Indexation</th>
<th>Distribution of Contract Durations</th>
<th>Mean Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega_1$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td>Random-Duration</td>
<td>0.00</td>
<td>0.42</td>
</tr>
<tr>
<td>Contracts</td>
<td>[0.25]</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Fixed-Duration</td>
<td>0.00</td>
<td>0.17</td>
</tr>
<tr>
<td>Contracts</td>
<td>[0.24]</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Random-Duration</td>
<td>0.86</td>
<td>0.98</td>
</tr>
<tr>
<td>Contracts</td>
<td>[0.24]</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Fixed-Duration</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>Contracts</td>
<td>[0.24]</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

Note: This table reports the estimated degree of dynamic indexation ($\delta$) and the distribution of contract durations ($\omega_j$) for each specification of the generalized price-setting framework (that is, random or fixed durations), obtained when the model is estimated using the benchmark markup gap and the demeaned level of inflation for each sample. For the German sample, the upper bound of the 95th percentile for the indexation parameter is enclosed in square brackets, while estimated standard errors of the other estimates are enclosed in parentheses. For the U.S. sample, confidence bounds and standard errors have not been computed because the estimated parameter vector is on the boundary of the admissible region; that is, the estimation algorithm imposes a maximum of 0.98 for the value of $\omega_1$.

Table 5 and the overidentification test results given in Table 6.
Table 6: Tests of Overidentifying Restrictions in the Absence of a Time-Varying Inflation Objective

<table>
<thead>
<tr>
<th></th>
<th>Random-Duration Contracts</th>
<th>Fixed-Duration Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany, 1975-1998</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
<td>United States, 1983-2003</td>
<td>0.004</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Note: This table indicates the probability that the overidentifying restrictions are consistent with each specification of the generalized price-setting framework, estimated using the benchmark markup gap and the demeaned level of inflation for each sample.

The setting framework provides a reasonable representation of the German data even in the absence of deterministic indexation.

Specifically, as for the German results reported in Section 5, the dynamic indexation parameter has a point estimate of zero and a value of only 0.25 for the upper bound of the 95 percent confidence interval. The mean duration of price contracts is noticeably longer (nearly three quarters for the random-duration specification and a bit longer for the fixed-duration specification), mainly due to the higher proportion of contracts with a four-quarter duration. While not shown in Table 5, the estimated degree of real rigidity is somewhat higher than that reported above: $\hat{\gamma} = 0.041$ for the random-duration case and 0.032 for the fixed-duration case, with estimated standard errors of 0.004 and 0.003, respectively.

In contrast, for the U.S. sample, the results are dramatically different when we ignore the possibility of a break in mean inflation in the early 1990s. In this case, the vector of parameter estimates is on the boundary of the admissible region of our estimation algorithm: $\hat{\omega}_1 = 0.98$, implying that virtually all contracts last only a single quarter; and $\hat{\gamma}$ is less than 0.001, implying that new prices exhibit virtually no responsiveness to real marginal cost. Of course, these estimates might be interpreted as supporting the view that U.S. prices are subject to very frequent adjustment and do not exhibit a substantial degree of nominal rigidity. Nevertheless, the overidentifying restrictions are decisively rejected in this case, thereby highlighting the importance of accounting for the time variation in the central

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35We conducted an extensive grid search to confirm that this vector of parameter estimates is indeed the global solution to the minimization problem. Because this vector lies on the boundary of the parameter space, we do not compute any standard errors or confidence intervals.
Table 7: Indirect Inference and the Hybrid NKPC

<table>
<thead>
<tr>
<th></th>
<th>Dynamic Slope Test of Overidentifying Indexation Coefficient Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Inflation Objective</td>
<td>0.47 (0.05) 0.004 (0.001) 0.000</td>
</tr>
<tr>
<td>Break in Inflation Objective</td>
<td>0.16 (0.07) 0.001 (0.001) 0.001</td>
</tr>
</tbody>
</table>

Note: This table reports the results of indirect inference regarding the degree of dynamic indexation ($\delta$) and the markup gap coefficient of the hybrid New Keynesian Phillips curve (NKPC), estimated using U.S. data over the sample period 1983–2003 under alternative assumptions about whether the inflation objective was constant over this period or exhibited a downward shift in 1991Q4. The standard error of each estimate is given in parentheses, while the final column indicates the p-value of the test of overidentifying restrictions.

bank’s inflation objective.

8 Reconsidering the Hybrid NKPC

As we have seen, the generalized price-setting framework does not yield plausible parameter estimates or reasonable goodness-of-fit for the U.S. sample (1983–2003) unless we account for the downward shift in the inflation rate that occurred during the early 1990s. At first glance, this outcome appears somewhat puzzling; after all, numerous other studies have analyzed similar samples of U.S. data and have obtained reasonable estimates of the hybrid NKPC without incorporating any shifts in the mean of inflation. Because the hybrid NKPC is a special case of our price-setting framework (as can be seen from equation (14)), we can shed further light on these issues by using our indirect inference procedures to estimate the hybrid NKPC and then compare the results with those discussed above.

As shown in Table 7, we obtain parameter estimates for the hybrid NKPC that are broadly in line with those obtained in previous studies. Under the assumption of a constant inflation objective, the indexation parameter $\delta$ is close to 0.5, while the low coefficient on the markup gap is consistent with the presence of a combination of nominal and real rigidities in price-setting behavior. Furthermore, once we allow for a break in the inflation objective
### Table 8: VAR Coefficients Implied by the Hybrid NKPC  

<table>
<thead>
<tr>
<th></th>
<th>First-Order Lags</th>
<th></th>
<th></th>
<th>Second-Order Lags</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inflation Measure</td>
<td>Markup Gap</td>
<td>Output Gap</td>
<td>Inflation Measure</td>
<td>Markup Gap</td>
<td>Output Gap</td>
</tr>
<tr>
<td><strong>Constant Inflation Objective</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid NKPC</td>
<td>0.44</td>
<td>0.12</td>
<td>0.14</td>
<td>-0.02**</td>
<td>0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>Unconstrained VAR</td>
<td>0.26</td>
<td>0.32</td>
<td>0.41</td>
<td>0.29</td>
<td>-0.27</td>
<td>-0.43</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.16)</td>
<td>(0.18)</td>
<td>(0.10)</td>
<td>(0.24)</td>
<td>(0.31)</td>
</tr>
<tr>
<td><strong>Break in Inflation Objective</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid NKPC</td>
<td>0.15</td>
<td>0.01*</td>
<td>0.00*</td>
<td>-0.02**</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Unconstrained VAR</td>
<td>0.20</td>
<td>0.29</td>
<td>0.31</td>
<td>0.25</td>
<td>-0.26</td>
<td>-0.38</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.15)</td>
<td>(0.19)</td>
<td>(0.10)</td>
<td>(0.22)</td>
<td>(0.31)</td>
</tr>
</tbody>
</table>

Note: This table indicates the coefficients (up to lag order 2) of the reduced-form VAR representation implied by the hybrid New Keynesian Phillips curve, along with the coefficients of the unconstrained VAR(3) estimated using U.S. data over the sample period 1983-2003. The standard errors of the unconstrained VAR coefficients are enclosed in parentheses. Asterisks indicate that the reduced-form VAR coefficient implied by the hybrid NKPC lies outside the 90 percent confidence interval (*) or 95 percent confidence interval (**) of the corresponding coefficient of the unconstrained VAR.

In the early 1990s, the degree of indexation drops to only 0.15 and is no longer statistically significant. Thus, as we have already emphasized, properly accounting for a time-varying inflation objective can have crucial implications for assessing the degree of forward-looking vs. backward-looking behavior in aggregate inflation dynamics.

This table also reveals that the overidentifying restrictions of the hybrid NKPC are decisively rejected by the data, with a p-value far below 1 percent regardless of whether or not the model incorporates a shift in the inflation objective. The source of these rejections is evident in Table 8, which compares the coefficients of the reduced-form VAR implied by the hybrid NKPC with the corresponding coefficients of the unconstrained VAR. In particular, while dynamic indexation is helpful in matching the first-order autoregressive properties of U.S. inflation, the hybrid NKPC completely fails to generate the second-order dynamics that are evident in the unconstrained VAR. By contrast, as previously shown in
Table 3, the overidentifying restrictions are *not* rejected for the generalized price-setting model once we account for the downward shift in the inflation objective.

9 Conclusion

In this paper, we have formulated a generalized price-setting framework that incorporates staggered contracts of multiple durations and that directly identifies the influences of nominal vs. real rigidities. In estimating this framework, we consider two distinct samples: German data for 1975-1998, and U.S. data for 1983-2003. For both samples, we find that the data is well-characterized by a contract distribution with an average duration of about two to three quarters and with a relatively high degree of real rigidity. Finally, our results indicate that backward-looking price-setting behavior is not needed to explain the aggregate data, at least in an environment with a stable monetary policy regime and a transparent and credible inflation objective.

This paper has proceeded under the assumption that all firms face the same output elasticity of marginal cost. In subsequent work, it will be interesting to explore whether this parameter varies systematically across groups of firms with different contract durations; that is, whether the aggregate data imply a cross-sectional relationship between nominal and real rigidities. Furthermore, the approach used here can easily be applied to other economies, especially for sample periods over which the inflation objective has been reasonably stable or has evolved gradually in a transparent way. Finally, our approach can be extended to consider the joint determination of aggregate wages and prices, in a framework that allows for multiple-period durations of both types of contracts.
References


